Quiz 6, February 15, 2012
Introduction to Probability - MATH/STATS 425, Winter 2012

N balls are put into M boxes independently at random (i.e. each ball is put independently in a randomly chosen box.) Find the expected number of empty boxes.

(Hint: express the number of empty boxes as a sum of M indicators.)

Let X = # of empty boxes. We can express

$$X = \sum_{i=1}^{M} X_i \quad \text{where} \quad X_i = \begin{cases} 1 & \text{if box } i \text{ is empty} \\ 0 & \text{otherwise.} \end{cases}$$

Then $E[X] = \sum_{i=1}^{M} E[X_i]$.

$E[X_i] = P\{X_i = 1\} = P\{\text{box } i \text{ is empty}\} = P\{\text{each of the } N \text{ balls is not put in box } i\}$

$= \left(1 - \frac{1}{M}\right)^N$ by independence ($\frac{1}{M}$ is the prob. to put a ball in box i)

Hence $E[X] = \boxed{M\left(1 - \frac{1}{M}\right)^N}$.