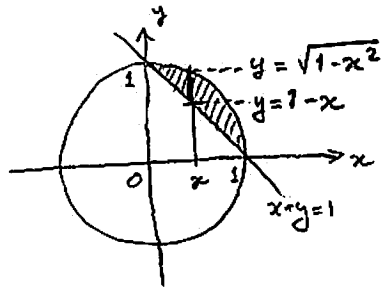
SOLUTIONS

Quiz 8, March 14, 2012
Introduction to Probability - MATH/STATS 425, Winter 2012

Suppose the joint distribution of random variables X and Y is given by the joint pdf

$$f(x,y) = \begin{cases} 2y/x & \text{if } x^2+y^2 \le 1,\ x+y \ge 1; \\ 0 & \text{elsewhere.} \end{cases}$$

Compute the marginal pdf of X.



$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_{1-x}^{\sqrt{1-x^2}} \frac{2y}{x}\,dy$$

$$= \frac{y^2}{x}\Big|_{y=1-x}^{\sqrt{1-x^2}} = \frac{1-x^2-(1-x)^2}{x}$$

$$= \frac{\cancel{1}-x^2-\cancel{1}+2x-x^2}{x} = 2(1-x), \quad \text{if} \quad 0 \le x \le 1.$$

Ans: $$f_X(x) = \begin{cases} 2(1-x) & \text{if } 0 \le x \le 1 \\ 0 & \text{elsewhere.} \end{cases}$$