

SOLUTIONS

Quiz 9, March 21, 2012

Introduction to Probability - MATH/STATS 425, Fall 2012

Let X and Y be random variables with joint pdf

$$f(x, y) = \begin{cases} 6x & \text{if } x > 0, y > 0 \text{ and } x + y < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Compute $P\{Y > X\}$.

$$\begin{aligned} P\{Y > X\} &= \iint_A 6x \, dx \, dy = \int_0^{1/2} \left(\int_x^{1-x} 6x \, dy \right) dx \\ &= \int_0^{1/2} 6x \left(\int_x^{1-x} dy \right) dx = \int_0^{1/2} 6x(1-2x) dx \\ &= \int_0^{1/2} (6x - 12x^2) dx = (3x^2 - 4x^3) \Big|_0^{1/2} \\ &= \frac{3}{4} - \frac{4}{8} = \boxed{\frac{1}{4}}. \end{aligned}$$

