Homework 1 (Answers)

(1) (a) ... rref contains row $[0\ 0\ 0\ |\ 1]$ $\Rightarrow$ no solutions

(12) (b) Unique solution $x=y=z=0$.

(c) Unique solution $x=4,\ y=8,\ z=5,\ t=2$.

(d) Unique solution $x=-3,\ y=-5,\ z=6,\ t=-3$.

(2) (a) Inconsistent

(c) $\begin{cases} x=-5-3y \\ y \text{ is free} \\ z=3 \end{cases}$ $\xrightarrow{y=t}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5-3t \\ t \\ 3 \end{bmatrix}$, $t \in \mathbb{R}$

Solution set is a line.

(3) $\begin{cases} x-4y=1 \\ 2x-y=-3 \\ x+3y=0 \end{cases}$ $\begin{bmatrix} \textcircled{1} & -4 & 1 \\ 2 & -1 & -3 \\ 1 & 3 & 0 \end{bmatrix} \begin{matrix} \\ -2\cdot I \\ -I \end{matrix} \rightarrow \begin{bmatrix} 1 & -4 & 1 \\ 0 & \textcircled{7} & -5 \\ 0 & 7 & -1 \end{bmatrix} -II \rightarrow \begin{bmatrix} 1 & -4 & 1 \\ 0 & \textcircled{7} & -5 \\ 0 & 0 & 4 \end{bmatrix} \begin{matrix} \\ \cdot 1/7 \\ \cdot 1/4 \end{matrix}$

(4) $\rightarrow \begin{bmatrix} \textcircled{1} & -4 & 1 \\ 0 & \textcircled{1} & -5/7 \\ 0 & 0 & 1 \end{bmatrix} \begin{matrix} \\ +4\cdot II \\ \\ \end{matrix} \longrightarrow \begin{bmatrix} 1 & 0 & \frac{27}{7} \\ 0 & 1 & -5/7 \\ 0 & 0 & 1 \end{bmatrix}$.

Inconsistent

$\Downarrow$

no common point.

(4) $x_1 \begin{bmatrix} 3 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} 6 \\ 7 \end{bmatrix} = \begin{bmatrix} -3 \\ 10 \end{bmatrix}$.

(10) $\begin{cases} 3x_1+6x_2=-3 \\ 5x_1+7x_2=10 \end{cases}$ $\begin{bmatrix} 3 & 6 & -3 \\ 5 & 7 & 10 \end{bmatrix} \frac{1}{3} \rightarrow \begin{bmatrix} \textcircled{1} & 2 & -1 \\ 5 & 7 & 10 \end{bmatrix} \xrightarrow[-5\cdot I]{} \begin{bmatrix} 1 & 2 & -1 \\ 0 & \textcircled{-3} & 15 \end{bmatrix} \cdot (-\frac{1}{3})$

$\rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 0 & \textcircled{1} & -5 \end{bmatrix} \begin{matrix} \\ -2\cdot II \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & 9 \\ 0 & 1 & -5 \end{bmatrix} \rightarrow \begin{matrix} x_1=9 \\ x_2=-5 \end{matrix}$

$\Rightarrow \boxed{\begin{bmatrix} -3 \\ 10 \end{bmatrix} = 9 \begin{bmatrix} 3 \\ 5 \end{bmatrix} - 5 \begin{bmatrix} 6 \\ 7 \end{bmatrix}}$.

(5) (10)
$$\left[\begin{array}{cc|c} 1 & h & 2 \\ 4 & 8 & k \end{array}\right] \begin{matrix} \\ -4\cdot I \end{matrix} \rightarrow \left[\begin{array}{cc|c} 1 & h & 2 \\ 0 & 8-4h & k-8 \end{array}\right]$$

Case 1: $8-4h=0 \Leftrightarrow \underline{h=2}$ Then

$$\rightarrow \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 0 & k-8 \end{array}\right]$$

If $k=8$ then $\rightarrow \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 0 & 0 \end{array}\right]$

∞ many solutions (x_2 is free)

If $k\neq 8$ then $\rightarrow \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 0 & 1 \end{array}\right]$
$\underline{II} \div (k-8)$

inconsistent due to row $[0\,0\,|\,1]$

Case 2: $8-4h \neq 0$. $(h\neq 2)$

$$\underline{II} \div (8-4h) \Rightarrow \left[\begin{array}{cc|c} 1 & h & 2 \\ 0 & 1 & \frac{k-8}{8-4h} \end{array}\right] \begin{matrix} \\ -h\cdot \underline{II} \end{matrix} \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 2-\frac{h(k-8)}{8-4h} \\ 0 & 1 & \frac{k-8}{8-4h} \end{array}\right]$$

$\Rightarrow$ unique solution.

Answer:
- If $h\neq 2$ then solution is unique.
- If $h=2$ and $k=8$ then there are infinitely many solutions
- If $h=2$ and $k\neq 8$ then there are no solutions.