Homework 1

Math 419, Winter 2013

1. In each exercise (a)-(d), the augmented matrix of a linear system is given. Describe the set of solutions of each system. Explain your answers.

(a) $\left[\begin{array}{ccc|c} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} 1 & -4 & 9 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right]$

(c) $\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 2 & 4 \end{array}\right]$ (d) $\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -2 \\ 0 & 1 & 0 & -4 & 7 \\ 0 & 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 & -3 \end{array}\right]$

2. Solve the following systems of equations. (Find the unique solution, explain why the system is inconsistent, or give the general form of a solution if there are infinitely many.)

(a)

$$\begin{aligned} x_1 - 3x_2 + 4x_3 &= -4 \\ 3x_1 - 7x_2 + 7x_3 &= -8 \\ -4x_1 + 6x_2 - x_3 &= 7 \end{aligned}$$

(b)

$$\begin{aligned} x_1 + 3x_3 &= 2 \\ x_2 - 3x_4 &= 3 \\ -2x_2 + 3x_3 + 2x_4 &= 1 \\ 3x_1 + 7x_4 &= -5 \end{aligned}$$

(c)

$$\begin{aligned} x + 3y + 4z &= 7 \\ 3x + 9y + 7z &= 6 \end{aligned}$$

3. Do the three lines $x - 4y = 1$, $2x - y = -3$ and $x + 3y = 0$ have a common point of intersection? Do not make a sketch.

4. Express the vector $\begin{bmatrix} -3 \\ 10 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 7 \end{bmatrix}$.

(Over, please)

5. Describe the values of h and k for which the system

$$x_1 + hx_2 = 2$$
$$4x_1 + 8x_2 = k$$

(i) has no solution;

(ii) has a unique solution;

(iii) has infinitely many solutions.

(There are separate answers for each part.)