Homework 10

Math 419, Winter 2013

1. Solve the following linear systems. If the solution is unique, find it **using Cramer's rule**. If the solution is not unique, find all solutions. If the solution does not exist, explain why.

(a)

$$\begin{aligned} -2x_1 + 3x_2 - x_3 &= 1 \\ x_1 + 2x_2 - x_3 &= 4 \\ -2x_1 - x_2 + x_3 &= -3 \end{aligned}$$

(b)

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 0 \\ x_2 - 3x_3 &= 0 \\ -x_1 + x_2 - x_3 &= 0 \end{aligned}$$

2. Compute the adjoint of A and the determinant of A, when

$$A = \begin{bmatrix} 1 & 0 & 5 \\ 2 & 1 & 0 \\ 0 & 4 & 0 \end{bmatrix}$$

3. Suppose a square matrix A has eigenvalues $\vec{v}_1, \ldots, \vec{v}_n$ and corresponding eigenvalues $\lambda_1, \ldots, \lambda_n$. Find the eigenvectors and eigenvalues of each of the following matrices.

(a) A^5.

(b) A^{-1}, assuming A is invertible.

(c) $A + 3I_n$.

(d) $10A$.

(e) $A^2 + 3A - 5I_n$.

4. Find all eigenvalues of the following matrices.

(a) $\begin{bmatrix} 2 & 7 \\ 7 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 3 \\ -4 & 4 \end{bmatrix}$

(c) The $n \times n$ matrix whose all entries equal 1.

5. Is the following fact true or false? If the determinant of a 2×2 matrix is non-positive, then A has at least one real eigenvalue.