Homework 10
Math 419, Winter 2013

1. Solve the following linear systems. If the solution is unique, find it using Cramer’s rule. If the solution is not unique, find all solutions. If the solution does not exist, explain why.

(a)

\[-2x_1 + 3x_2 - x_3 = 1
\]
\[x_1 + 2x_2 - x_3 = 4
\]
\[-2x_1 - x_2 + x_3 = -3
\]

(b)

\[x_1 + 2x_2 + x_3 = 0
\]
\[x_2 - 3x_3 = 0
\]
\[-x_1 + x_2 - x_3 = 0
\]

2. Compute the adjoint of $A$ and the determinant of $A$, when

\[
A = \begin{bmatrix}
1 & 0 & 5 \\
2 & 1 & 0 \\
0 & 4 & 0 \\
\end{bmatrix}
\]

3. Suppose a square matrix $A$ has eigenvalues $\vec{v}_1, \ldots, \vec{v}_n$ and corresponding eigenvalues $\lambda_1, \ldots, \lambda_n$. Find the eigenvectors and eigenvalues of each of the following matrices.

(a) $A^5$.

(b) $A^{-1}$, assuming $A$ is invertible.

(c) $A + 3I_n$.

(d) $10A$.

(e) $A^2 + 3A - 5I_n$.

4. Find all eigenvalues of the following matrices.

(a) \[
\begin{bmatrix}
2 & 7 \\
7 & 2 \\
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
5 & 3 \\
-4 & 4 \\
\end{bmatrix}
\]

(c) The $n \times n$ matrix whose all entries equal 1.

5. Is the following fact true or false? If the determinant of a $2 \times 2$ matrix is non-positive, then $A$ has at least one real eigenvalue.