Homework 11

Math 419, Winter 2013

1. Does there exist a 4×4 matrix without real eigenvalues? Give en example or prove it does not exist.

2. Let A be a 2×2 matrix with $\operatorname{tr}(A) = 7$ and $\det(A) = 12$.

(a) Determine the eigenvalues of A.

(b) Is matrix A with these properties unique? Either prove it is unique or give an example of two different matrices as above.

3. Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 3 \end{bmatrix}$$

For what values of a, b, c does there exists a basis of eigenvectors of A?

4. Suppose that there is a basis of eigenvectors of an $n \times n$ matrix A. What is the relationship between geometric and algebraic multiplicities of A?

5. Diagonalize the following matrices. If the matrix is diagonalizable, compute the diagonal matrix D and the matrix S^{-1} (or S, whichever you like) such that $A = S^{-1}DS$. If the matrix is not diagonalizable, explain why.

(a) $A = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$

(d) $A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 1 & 4 & -3 & 0 \\ -1 & -2 & 0 & -3 \end{bmatrix}$

6. Determine whether the following statement is true or false. Justify. If an invertible matrix A can be diagonalized, then A^{-1} can be diagonalized, too.