Homework 12

Math 419, Winter 2013

1. Let
$$A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}.$$
Diagonalize A and use this to compute
$$A^3 - 4A^2 + 2A - 9I.$$
Show all steps.

2. Find the eigenvalues and eigenvectors of the following matrices. Diagonalize these matrices. Show all steps.

(a) $\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 1 \\ -8 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & -5 \\ 1 & 3 \end{bmatrix}$

3. Consider vectors $\vec{u}, \vec{v} \in \mathbb{R}^n$ such that $\|\vec{u} + \vec{v}\| = 3$ and $\|\vec{u} - \vec{v}\| = 1$. Determine $\vec{u} \cdot \vec{v}$.

Hint: simplify the expression $\|\vec{u} + \vec{v}\|^2 - \|\vec{u} - \vec{v}\|^2$.

4. Compute the spectral decomposition for the following matrices:

(a) $\begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$ To save you time, the eigenvalues are $7, 7, -2$.

(c) $\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$ To save you time, the eigenvalues are $5, 2, 2$.

5. Mark each statement True or False. Justify.

(a) For a nonzero vector $\vec{v} \in \mathbb{R}^n$, the matrix $\vec{v}\,\vec{v}^{\mathsf{T}}$ has rank $= 1$.

(b) For a nonzero vector $\vec{v} \in \mathbb{R}^n$, the matrix $\vec{v}\,\vec{v}^{\mathsf{T}}$ is the matrix of an orthogonal projection onto $\text{span}(\vec{v})$.

(c) If a real matrix A can be orthogonally diagonaled (i.e. if $A = S^{-1}DS$ for some orthogonal matrix S and a diagonal matrix D), then A is symmetric.

(e) Similar matrices have the same rank.

(f) Two matrices with the same trace and same determinant must be similar.

(g) All invertible matrices can be diagonalized (over $\mathbb{C}$).