Homework 13

Math 419, Winter 2013

1. Diagonalize the following quadratic forms. That is, find a change of variable $\vec{y} = U\vec{x}$ so that the quadratic form becomes canonical (without cross-product terms). Determine whether each quadratic form is positive definite, positive semidefinite, or neither.

(a) $5x_1^2 - 4x_1x_2 + 5x_2^2$

(b) $8x_1^2 + 6x_1x_2$

2. For each matrix, find singular values and singular vectors (right and left). Find a singular value decomposition. Show all steps.

(a) $\begin{bmatrix} -3 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$

3. Let A be a square matrix, Show that $|\det A|$ is the product of the singular values of A.

4. Let A be the rotation in the plane by angle $\pi/4$ counter-clockwise. Find the singular values and singular vectors (left and right) of A.

5. Mark each statement True or False. Justify.

(a) Any matrix of rank r can be expressed as a sum of r matrices of rank 1.

(b) If all singular values of A equal 1 then A is orthogonal.

(c) If A is orthogonal then all singular values of A equal 1.

(d) The ranks of $A^{\mathsf{T}}A$ and AA^{T} are equal.

(e) The eigenvalues of a symmetric matrix A are the same as singular values of A.