Homework 2 Solutions.

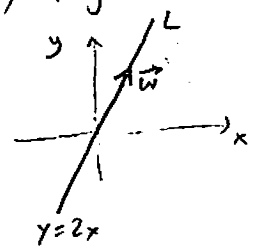
(1)

a) Reflection. $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (5)

Rotation: $B = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \end{bmatrix}$

Composition: $BA = \begin{bmatrix} \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} \end{bmatrix}$.

(b) Projection: (5)



$\vec{w} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \in$ line. $\|\vec{w}\| = \sqrt{1^2+2^2} = \sqrt5$

$\vec{u} = \frac{\vec{w}}{\|\vec{w}\|} = \begin{bmatrix} 1/\sqrt5 \\ 2/\sqrt5 \end{bmatrix}$.

$$P = \begin{array}{c c} & \begin{array}{cc} 1/\sqrt5 & 2/\sqrt5 \end{array} \\ \begin{array}{c} \frac{1}{\sqrt5} \\ \frac{2}{\sqrt5} \end{array} & \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & 4/5 \end{bmatrix} \end{array}.$$

Rotation B as in (a).

Composition: $BP = \begin{bmatrix} \frac{1}{\sqrt2} & -\frac{1}{\sqrt2} \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \end{bmatrix} \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & 4/5 \end{bmatrix} = \begin{bmatrix} -\frac{1}{5\sqrt2} & -\frac{2}{5\sqrt2} \\ \frac{3}{5\sqrt2} & \frac{6}{5\sqrt2} \end{bmatrix}$

(2) $AB = \begin{bmatrix} -7 & 18+3k \\ -4 & -9+k \end{bmatrix}$, $BA = \begin{bmatrix} -7 & 12 \\ -6-k & -9+k \end{bmatrix}$.

$AB = BA \iff \begin{cases} 18+3k = 12 \\ -4 = -6-k \end{cases} \iff k = -2$.

(3) Different answers are possible, e.g. (10)

(a) $B = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. (Such matrices are called <u>nilpotent</u>)

$\exists k: A^k = 0$

(4) For $\vec{u} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $T(\vec{u}+\vec{v}) \neq T(\vec{u}) + T(\vec{v})$

Other reasons are possible, too.

(5) $AD = \begin{bmatrix} d_1 & 2d_2 & 3d_3 \\ 4d_1 & 5d_2 & 6d_3 \\ 7d_1 & 8d_2 & 9d_3 \end{bmatrix}$, $DA = \begin{bmatrix} d_1 & 2d_1 & 3d_1 \\ 4d_2 & 5d_2 & 6d_2 \\ 7d_3 & 8d_3 & 9d_3 \end{bmatrix}$.

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- Multiplying A by D on the right amounts to multiplying the columns of A by d_i
- Multiplying on the left amounts to multiplying the rows by d_i

(6) $\begin{cases} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \\ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \end{bmatrix} \end{cases}$ $\begin{cases} a_{11} + a_{12} = 3 \\ a_{21} + a_{22} = -1 \\ a_{11} - a_{12} = -1 \\ a_{21} - a_{22} = -3 \end{cases}$

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Solving for $a_{11}, a_{12}, a_{21}, a_{22}$ yields: $\begin{cases} a_{11} = 1, & a_{12} = 2, \\ a_{21} = -2, & a_{22} = 1 \end{cases}$

$\Rightarrow A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$.