

Homework 2

Math 419, Winter 2013

1. In each part below, find the matrix of the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that:

(a) First reflects points about the x -axis and then rotates by $\pi/4$ radians counterclockwise;

(b) First projects points onto the line $y = 2x$ and then rotates by $\pi/4$ radians counterclockwise;

(c) First reflects points about the line $y = 3x$ and then dilates in the horizontal direction by the factor 2

2. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 9 \\ -3 & k \end{bmatrix}$. What value(s) of k , if any, will make $AB = BA$?

3. (a) Let $A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$. Construct a non-zero 2×2 matrix B such that $AB = 0$ (the matrix whose all entries are zero).

(b) Find an example of a non-zero matrix A for which $A^2 = 0$. (Here $A^2 = AA$.)

4. Consider a transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined as

$$T(x, y) = (x + 1, 2y).$$

Prove that T is not a linear transformation.

5. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}.$$

Compute AD and DA . Describe in words the effect of multiplication by a diagonal matrix D on the right and on the left.

6. Find the (matrix of the) linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that transforms vector $(1, 1)$ into vector $(3, -1)$ and vector $(1, -1)$ into vector $(-1, -3)$.