Homework 3

Math 419, Winter 2013

1. For each matrix below, compute its inverse or explain why it is not invertible. Show all steps.

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{bmatrix}$

2. Suppose $AB = AC$ where A is an $n \times n$ matrix, B and C are $n \times m$ matrices. Show that $B = C$. Is this true, in general, when A is not invertible? (Prove this or give an example where it fails).

3. Consider the $n \times n$ matrix of the form

$$A = \begin{bmatrix} I_{n-1} & \vec{v} \\ \vec{u} & 1 \end{bmatrix}.$$

(The upper-left $(n-1) \times (n-1)$ block of A is the identity matrix; the lower-right entry is 1; $\vec{u} \in \mathbb{R}^{n-1}$ is a row-vector and $\vec{v} \in \mathbb{R}^{n-1}$ is a column vector. I suggest to write this matrix for $n = 4$ to see more clearly what is going on.)

Describe when A is invertible, in terms of $\vec{u}$ and $\vec{v}$.

4. For each matrix below, compute its image and kernel.

(a) $\begin{bmatrix} 3 & -4 \\ -9 & 12 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & 1 \\ 3 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 1 & 0 \\ 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -3 \\ -1 & 3 \\ 2 & 6 \end{bmatrix}$

5. Find an example of a 2×2 matrix such that $\operatorname{im}(A) = \ker(A)$.