Homework 4

Math 419, Winter 2013

1. (a) Find a 3×3 matrix A whose image is the line that passes through points $(0,0,0)$ and $(1,-1,1)$.

(b) Find a 3×3 matrix A whose kernel is the xy-plane and whose image is the z-axis.

(c) Find a 2×2 matrix A whose image is the line $y = x$ and whose kernel is the x-axis.

2. Let A be a $p \times m$ matrix and B be an $n \times p$ matrix. In each part, determine if there is any relationship between the two given sets. Are they necessarily equal? Is one of them always contained in the other? Explain.

(a) $\ker(BA)$ and $\ker(A)$.

(b) $\mathrm{im}(BA)$ and $\mathrm{im}(A)$.

3. For each set of vectors, determine whether it is linearly independent, whether it spans $\mathbb{R}^3$, and whether it forms a basis of $\mathbb{R}^3$.

(a) $\vec{v}_1 = (3,0,6)$, $\vec{v}_2 = (-4,1,7)$, $\vec{v}_3 = (-2,1,5)$.

(b) $\vec{v}_1 = (1,1,1)$, $\vec{v}_2 = (1,1,0)$, $\vec{v}_3 = (0,0,1)$.

(c) $\vec{v}_1 = (1,1,1)$, $\vec{v}_2 = (0,0,0)$, $\vec{v}_3 = (0,1,1)$.

(d) $\vec{v}_1 = (1,0,-3)$, $\vec{v}_2 = (3,1,-4)$, $\vec{v}_3 = (-2,-1,1)$.

(d) $\vec{v}_1 = (1,2,-4)$, $\vec{v}_2 = (-4,3,6)$.

(a) $\vec{v}_1 = (3,0,6)$, $\vec{v}_2 = (-4,1,7)$, $\vec{v}_3 = (-2,1,5)$, $\vec{v}_4 = (3,1,2)$. (Note that the first three vectors are as in (a).)

4. (a) Find a basis of the plane $x - 3y + 2z = 0$ in $\mathbb{R}^3$.

(b) Find the representation of the vector $(6,2,0)$ as a linear combination of the basis vectors from (a).

5. Are the following sets linear subspaces? Explain.

(a) $W \subset \mathbb{R}^3$ consisting of all vectors of the form $(a,b,1)$ where a and b are real numbers.

(b) $W \subset \mathbb{R}^4$ consisting of all vectors of the form $(a,b,a-b,a+2b)$ where a and b are real numbers.

(b) $W \subset \mathbb{R}^3$ consisting of all vectors of the form (a,b,ab) where a and b are real numbers.