Homework 5

Math 419, Winter 2013

1. In each part, find examples of matrices as stated, or explain why no such examples exist:

(a) 3×7 matrix with $\dim(\ker(A)) = 4$.

(b) 8×3 matrix with $\dim(\ker(A)) = 4$.

(c) 4×6 matrix with $\dim(\ker(A)) = 4$ and $\operatorname{rank}(A) = 3$

(d) 4×6 matrix with $\dim(\ker(A)) = 5$ and $\operatorname{rank}(A) = 1$.

(e) An invertible 3×3 matrix C which can be expressed as $C = BA$ for some 5×3 matrix A and 3×5 matrix B.

2. Which of the following sets are linear spaces? If you believe that it is a linear space, do not check all the axioms – just check that the set is closed under addition and multiplication by scalars. If you believe that the set is *not* a linear space, explain why (find an example that violates one of the requirements).

(a) The set of all functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(0) = 1$

(b) The set of all functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(0) = 0$

(c) The set of all continuous functions $f : [0, 1] \to \mathbb{R}$ such that $\int_0^1 f(x)\, dx = 0$

(d) The set of all invertible 2×2 matrices

3. In each part, construct a basis for the linear space V, and compute the dimension of V.

(a) V is the set of all 2×2 matrices A satisfying $BA = 0$, where $B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

(b) V is the set of quadratic polynomials f such that $f(0) = 0$.

(c) V is the set of even quadratic polynomials f, i.e. such that $f(x) = f(-x)$ for all x.

4. Find the image, kernel and rank of the following linear transformations:

(a) $T(f) = \int_1^4 f(x)\, dx$ from $P_2[x]$ to $\mathbb{R}$.

(b) $T(f) = f(8)$ from $P_2[x]$ to $\mathbb{R}$.

(c) $T(f) = xf'(x)$ from $P_2[x]$ to $P_2[x]$. (For example, if $f(x) = x - x^2$ then $(T(f))(x) = x(1 - 2x) = x - 2x^2$.)

5. One basis for the linear space $P_2[x]$ is formed by the monomials 1, x, x^2. Construct a different basis for $P_2[x]$ whose elements are all quadratic functions (i.e. they are all of the form $a + bx + cx^2$ with $c \neq 0$.) Explain why your set is a basis.