Homework 6

Math 419, Winter 2013

1. In each part, determine if the given linear transformation is an isomorphism. Explain your reasoning.

(a) $T(f) = f - 2f'$ from $P_3[x]$ to $P_3[x]$

(b) An orthogonal projection in $\mathbb{R}^3$ onto the plane $x + y + z = 0$.

(c) $T(a, b, c) = (a - b, b - c, a - c)$ from $\mathbb{R}^3$ to $\mathbb{R}^3$

(d) $T(f) = f(t+1)$ from $C[1, 2]$ to $C[0, 1]$ (these are spaces of continuous functions).

2. In each part, determine if the given linear spaces V and W are isomorphic. If they are isomorphic, give an example of isomorphism $T : V \to W$. If they are not isomorphic, explain why.

(a) The plane V defined by the equation $x + y + z = 0$ and the plane W defined by the equation $x - 2y = 0$, both as subspaces of $\mathbb{R}^3$.

(b) The space V of quadratic polynomials of the form $a + cx^2$ and $W = \mathbb{R}^2$.

(c) The image V and kernel W of any given 3×3 matrix.

(d) Spaces of continuous functions $V = C[0, 1]$ and $V = C[0, 2]$.

3. (a) Find the change of basis matrix from the standard basis of $\mathbb{R}^2$ to the basis $\mathcal{B}$ consisting of the vectors $(-1, 2)$ and $(-3, 5)$.

(b) Compute the coordinates of the vector $(2, -5)$ in the basis $\mathcal{B}$, i.e. compute $[\vec{x}]_{\mathcal{B}}$ for that vector.

4. In each part, find the matrices of the given linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, first with respect to the standard basis of $\mathbb{R}^2$, and then with respect of the basis consisting of the vectors $(1, 3)$ and $(2, 5)$:

(a) $T(x, y) = (2y, 3x - y)$;

(b) $T(x, y) = (3x - 4y, x + 5y)$.

5. Compute the orthogonal projection in $\mathbb{R}^3$ of the vector $(-1, 1, 3)$ onto the plane spanned by the vectors $(2, 5, 2)$ and $(3, -2, 2)$. [Hint: are these two vectors orthogonal, orthonormal or neither?]

6. Among all unit vectors (x, y, z) in R^3, find the one for which the sum $x - 2y + 4z$ is maximal. [Hint: think about Cauchy-Schwarz inequality.]