1. In each part, determine if the given linear transformation is an isomorphism. Explain your reasoning.

(a) \( T(f) = f - 2f' \) from \( P_3[x] \) to \( P_3[x] \)

(b) An orthogonal projection in \( \mathbb{R}^3 \) onto the plane \( x + y + z = 0 \).

(c) \( T(a, b, c) = (a - b, b - c, a - c) \) from \( \mathbb{R}^3 \) to \( \mathbb{R}^3 \)

(d) \( T(f) = f(t+1) \) from \( C[1, 2] \) to \( C[0, 1] \) (these are spaces of continuous functions).

2. In each part, determine if the given linear spaces \( V \) and \( W \) are isomorphic. If they are isomorphic, give an example of isomorphism \( T: V \to W \). If they are not isomorphic, explain why.

(a) The plane \( V \) defined by the equation \( x + y + z = 0 \) and the plane \( W \) defined by the equation \( x - 2y = 0 \), both as subspaces of \( \mathbb{R}^3 \).

(b) The space \( V \) of quadratic polynomials of the form \( a + cx^2 \) and \( W = \mathbb{R}^2 \).

(c) The image \( V \) and kernel \( W \) of any given \( 3 \times 3 \) matrix.

(d) Spaces of continuous functions \( V = C[0, 1] \) and \( V = C[0, 2] \).

3. (a) Find the change of basis matrix from the standard basis of \( \mathbb{R}^2 \) to the basis \( B \) consisting of the vectors \((-1, 2)\) and \((-3, 5)\).

(b) Compute the coordinates of the vector \((2, -5)\) in the basis \( B \), i.e. compute \([\vec{x}]_B\) for that vector.

4. In each part, find the matrices of the given linear transformation \( T: \mathbb{R}^2 \to \mathbb{R}^2 \), first with respect to the standard basis of \( \mathbb{R}^2 \), and then with respect of the basis consisting of the vectors \((1, 3)\) and \((2, 5)\):

(a) \( T(x, y) = (2y, 3x - y) \);

(b) \( T(x, y) = (3x - 4y, x + 5y) \).

5. Compute the orthogonal projection in \( \mathbb{R}^3 \) of the vector \((-1, 1, 3)\) onto the plane spanned by the vectors \((2, 5, 2)\) and \((3, -2, 2)\). [Hint: are these two vectors orthogonal, orthonormal or neither?]

6. Among all unit vectors \((x, y, z)\) in \( \mathbb{R}^3 \), find the one for which the sum \( x - 2y + 4z \) is maximal. [Hint: think about Cauchy-Schwarz inequality.]