Homework 7

Math 419, Winter 2013

1. Find the matrix of the given linear transformation T in the given basis. Determine if T is an isomorphism.

(a) $T : P_2[x] \to P_2[x]$ is defined as $T(f) = f'' + 4f'$. The basis is $\{1, x, x^2\}$.

(b) $T : V \to V$, where $V = \text{span}\{\cos x, \sin x, 1\}$, is defined as $T(f) = f - 2f' + 3f''$. The basis is $\{\cos x, \sin x, 1\}$.

2. Consider the following vectors in $\mathbb{R}^4$:

$\vec{v}_1 = (1/2, 1/2, 1/2, 1/2)$, $\vec{v}_2 = (1/2, 1/2, -1/2, -1/2)$, $\vec{v}_3 = (1/2, -1/2, 1/2, -1/2)$.

Can you find a vector $\vec{v}_4$ in $\mathbb{R}^4$ so that the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ are orthonormal? If so, how many such vectors are there?

3. Two subspaces V and W of $\mathbb{R}^n$ are called *orthogonal subspaces* if $\vec{v} \cdot \vec{w} = 0$ for all $\vec{v} \in V$, $\vec{w} \in W$. Answer True or False for each part. Explain why if you choose True, and give a counter-example if you choose False.

(a) $\ker(A)$ and $\text{im}(A)$ are orthogonal for every $n \times n$ matrix A.

(b) There are infinitely many pairs of two planes in $\mathbb{R}^2$ which are orthogonal.

(c) Subspaces V and W are orthogonal if and only if $W = V^\perp$.

(e) If V and W are orthogonal, then $V^\perp$ and $W^\perp$ are orthogonal.

4. Find a basis for $V^\perp$ where V is the subspace of $\mathbb{R}^4$ spanned by the vectors $\vec{v}_1 = (1, 2, 3, 4)$ and $\vec{v}_2 = (5, 6, 7, 8)$.

5. Suppose vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5$ form an orthonormal basis of $\mathbb{R}^5$. Compute the length of the vector

$$\vec{v} = 3\vec{v}_1 - 2\vec{v}_2 + 7\vec{v}_3 - \vec{v}_4 + \vec{v}_5.$$