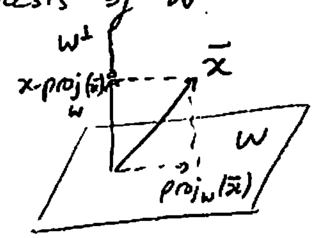
Homework 8 Solutions

(1) $\vec{u}_1 \cdot \vec{u}_2 = 0 \Rightarrow \vec{u}_1, \vec{u}_2$ are orthogonal 10

$\|\vec{u}_1\| = \sqrt{1^2+3^2+2^2} = \sqrt{14}$, $\quad \|\vec{u}_2\| = \sqrt{5^2+1^2+4^2} = \sqrt{42}$.

$\vec{V}_1 = \frac{\vec{u}_1}{\sqrt{14}}$, $\vec{V}_2 = \frac{\vec{u}_2}{\sqrt{42}}$ form an orthonormal basis of W.

$$\operatorname{proj}_W(\vec{x}) = (\vec{V}_1 \cdot \vec{x})\vec{V}_1 + (\vec{V}_2 \cdot \vec{x})\vec{V}_2 = \begin{bmatrix} 10/3 \\ 2/3 \\ 8/3 \end{bmatrix}$$



$$x - \operatorname{proj}_W(\vec{x}) = \begin{bmatrix} -7/3 \\ 7/3 \\ 7/3 \end{bmatrix}.$$

Answer.
$$\vec{x} = \underset{\substack{\uparrow \\ W}}{\begin{bmatrix} 10/3 \\ 2/3 \\ 8/3 \end{bmatrix}} + \underset{\substack{\uparrow \\ W^\perp}}{\begin{bmatrix} -7/3 \\ 7/3 \\ 7/3 \end{bmatrix}}$$

(2) A basis of the plane is $\vec{v}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{V}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$. 10

Gram-Schmidt process yields an orthonormal basis

$$\vec{u}_1 = \frac{\vec{V}_1}{\|\vec{V}_1\|} = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{u}_2 = \frac{\vec{V}_2^\perp}{\|\vec{V}_2^\perp\|} = \frac{\vec{V}_2 - (\vec{u}_1 \cdot \vec{V}_2)\vec{u}_1}{\| \text{—} \|} = \frac{1}{\sqrt{6}}\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

There are other correct solutions

(3) (a) $\begin{bmatrix} 2/\sqrt{30} \\ -5/\sqrt{30} \\ 1/\sqrt{30} \end{bmatrix}, \begin{bmatrix} 2/\sqrt{6} \\ 1/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$, (b) $\begin{bmatrix} 3/\sqrt{50} \\ -4/\sqrt{50} \\ 5/\sqrt{50} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$. 10

(4) (a) True: if $\vec{w} \in W$ then $\vec{w} = c_1\vec{u}_1 + c_2\vec{u}_2$ for some $c_1, c_2 \in \mathbb{R}$ 5

$\vec{z} \cdot \vec{w} = \vec{z}(c_1\vec{u}_1 + c_2\vec{u}_2) = c_1\,\vec{z}\cdot\vec{u}_1 + c_2\,\vec{z}\cdot\vec{u}_2 = 0 + 0 = 0$

$\Rightarrow \vec{z} \perp w$ Since this holds for all $w \in W \Rightarrow \vec{z} \in W^\perp$.

(b) False, by def of orthogonal projection.

(c) True, ~~eg~~ either by def. of orthogonal projection, or by the formula (using the ~~basis~~ orthonormal basis). 5