Homework 8

Math 419, Winter 2013

1. Let W be the subspace spanned by the vectors $\vec{u}_1$, $\vec{u}_2$ given below. Write vector $\vec{x}$ as the sum of a vector in W and a vector orthogonal to W.

$$\vec{x} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \quad \vec{u}_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}, \quad \vec{u}_3 = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}.$$

2. Find an orthonormal basis of the plane $x + y + z = 0$ (as a subspace of $\mathbb{R}^3$).

3. In each part, the given set is a basis for a subspace W. Use the Gram-Schmidt process to produce an orthonormal basis of W.

(a) $\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}.$

(b) $\begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 14 \\ -7 \end{bmatrix}.$

4. Mark each statement True or False, Justify each answer.

(a) If $\vec{z}$ is orthogonal to $\vec{u}_1$ and $\vec{u}_2$ and if $W = \text{span}(\vec{u}_1, \vec{u}_2)$ then $\vec{z}$ must be in $W^\perp$.

(b) The orthogonal projection $\text{proj}_V(\vec{x})$ of a vector $\vec{x}$ onto a subspace V can sometimes depend on the orthogonal basis for V used to compute $\text{proj}_V(\vec{x})$.

(c) If $\vec{x}$ is in a subspace V, then the orthogonal projection of $\vec{x}$ onto V is $\vec{x}$ itself.