

Homework 9

Math 419, Winter 2013

1. Mark each statement True or False. Justify each answer.

(a) If U and V are orthogonal matrices then UV is an orthogonal matrix.

(b) The set of all orthogonal 3×3 matrices forms a linear space.

(c) The determinant of all orthogonal 2×2 matrices is 1.

(d) All entries of an orthogonal matrix are less than or equal to 1.

(e) If A is an orthogonal matrix then A^T is orthogonal.

2. Find an orthogonal matrix A such that

$$A \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

3. In each part, find the vector \vec{x} which minimizes $\|A\vec{x} - \vec{b}\|$.

(a) $A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$.

(b) $A = \begin{bmatrix} -1 & 2 \\ 2 & -3 \\ -1 & 3 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$.

4. Compute the determinants of the following matrices. Indicate what method you use. Show all steps.

(a) $A = \begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{bmatrix}$

(c) $A = \begin{bmatrix} 3 & -7 & 8 & 9 & -6 \\ 0 & 2 & -5 & 7 & 3 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 2 & 4 & -1 \\ 0 & 0 & 0 & -2 & 0 \end{bmatrix}$.