MAT-127A-2 MIDTERM EXAM February 13, 2004

50 minutes, 4 problems, closed books, closed notes, no calculators

1. Give $\inf S$ and $\sup S$ (as real numbers or $+\infty$ or $-\infty$) for the following sets S:

(a) `[3 points]` $\{x \in \mathbb{Q} \ : \ x^4 < 1\}$

(b) `[5 points]` $\{x \in \mathbb{R} \ : \ \sin x > 0\}$

(c) `[7 points]` $\bigcap_{n=1}^{\infty} (-n, \frac{1}{n})$

2. Give the following if they exist (as real numbers or $+\infty$ or $-\infty$). Otherwise assert "NOT EXIST".

(a) `[5 points]` $\lim \frac{5n^2+(-1)^n}{3n^2+n+1}$

(b) `[5 points]` $\lim (n-10)^{1/3}$

(a) `[5 points]` $\lim \cos(\frac{\pi n}{4})$

3. Let $s_1 = 0$ and $s_{n+1} = \frac{1}{2}(s_n + 1)$ for $n \geq 1$.

(a) `[10 points]` Use induction to prove that $s_n \leq 1$ for all n.

(b) `[7 points]` Prove that (s_n) is a nondecreasing sequence.

(c) `[13 points]` Prove that $\lim s_n$ exists and find $\lim s_n$.

4. Let (s_n) and (t_n) be two convergent sequences and such that

$$s_n \leq t_n \quad \text{for all } n \in \mathbb{N}.$$

(a) `[25 points]` Prove that $\lim s_n \leq \lim t_n$. (*Hint: you may want to look at the sequence* $(t_n - s_n)$*, but you might come up with a direct proof as well)*

(b) `[15 points]` Will part (a) remain true if one replaces **both** inequality signs "$\leq$" above by the strict inequality signs "$<$"? (*Prove or give an example of sequences for which this fails. Just a correct guess will not earn any credit)*