Midterm Exam 1. Math 451, Fall 2015, Prof. Vershynin

1. *(10 points)* Let S be a subset of $\mathbb{R}$. Suppose S is bounded above, and some upper bound u is an element of S. Prove that $\sup S = u$.

2. *(10 points)* For each of the following statements, decide if it is true or false. Prove or give a counterexample.

(a) *(5 points)* There exists a sequence of irrational numbers which converges to a rational number.

(b) *(5 points)* There exists a sequence that has a bounded subsequence but has no convergent subsequences.

3. *(10 points)* Compute the limit

$$\lim \left(\sqrt{4n^2 + n} - 2n \right).$$

4. *(10 points)* Let (x_n) be a sequence that converges to a non-zero limit. Prove that all except finitely many terms x_n are non-zero.

5. *(10 points)* Let (x_n) be an increasing sequence and (y_n) be a decreasing sequence. Assume that $x_n \leq y_n$ for all n. Prove that both sequences converge.

6. *(10 points)* Prove that

$$\lim \frac{n!}{n^n} = 0.$$