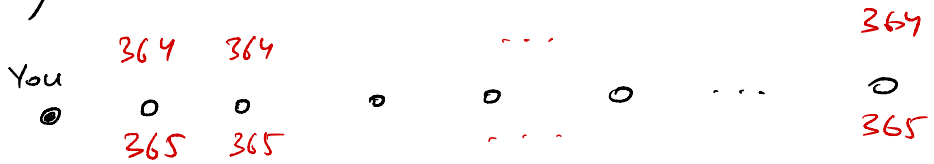
S1 : E1

Birthday Problem What is the probability that one of your classmates has the same BD as you?

↑ 60 students

Solution: Opposite ("complementary") question:

Probability that **no one** has your BD?



- # ways to assign BD's to students $= \underbrace{365 \times 365 \times \cdots \times 365}_{59 \text{ times}} = 365^{59}$
- # (ways to assign BD's so that no one has yours) $= \underbrace{364 \times 364 \times \cdots \times 364}_{59 \text{ times}} = 364^{59}$
- All assignments are equally likely $\Rightarrow$

$$P\{\text{no one has your BD}\} = \frac{364^{59}}{365^{59}} = \left(\frac{364}{365}\right)^{59} \qquad (*)$$

- $\Rightarrow P\{\text{someone has your BD}\} \approx 1 - \left(\frac{364}{365}\right)^{59} \approx \boxed{0.15}$, 15%.

Ex: If instead of 60 students there are 366 students,

$$P\{\text{no one shares your BD}\} = \left(\frac{364}{365}\right)^{365} = \left(1 - \frac{1}{365}\right)^{365} \approx \frac{1}{e} = \boxed{0.36}$$

$\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x$, $x = -1$

Q1: Why not

$P\{\text{someone shares your BD}\} = \underbrace{\frac{1}{365} + \frac{1}{365} + \cdots + \frac{1}{365}}_{59 \text{ students}} = \frac{59}{365} = 0.16$?

student 1, student 2, ...

Not disjoint

Q2: Why ratio in (*)? Later.