S1: E4

Recall S1: E2   A <u>permutation</u> of n objects $=$ $\forall$ ordered arrangement of them.

There are $1\cdot 2\cdot 3 \cdots n = n!$ permutations.

Ex How many different words can be made by rearranging the letters of the word DOCTOR?

$$\frac{6!}{2} = \frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}{2} = 360$$

all permutations

overcounting correction: ignoring the order of 2 O's

Ex Same problem for word SUCCESS?

$$\frac{7!}{3!\,2!} = 420$$

all permutation

ignoring the order of C's

ignoring the order of S's

Ex In how many ways can Alisa invite 3 from her 7 friends for her party?

Solution 1

The invitation list = word with 3 letters Y and 4 letter N:

invited ↓ Y   non-invited ↓ N

Friend	1	2	3	4	5	6	7
Invited?	N	Y	N	N	Y	N	Y

$$\# \text{ of such words} = \frac{7!}{3!\,4!} = 35.$$

ignoring the order of Y's and N's

Solution 2 :

$$\#(\text{ways to send invitations to 3 friends}) = \overset{\text{invite 1st}}{7} \cdot \overset{\text{2nd}}{6} \cdot \overset{\text{3rd}}{5}$$

Ignore the order of invitations $\Rightarrow$

$$\frac{7\cdot 6\cdot 5}{3!} = \frac{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{3!\cdot 4\cdot 3\cdot 2\cdot 1} = \boxed{\frac{7!}{3!4!}} = 35.$$

- More generally :

$$\#(\text{ways to choose } \underbrace{k}_{\text{e.g. } k \text{ friends}} \text{ objects from } \underbrace{n}_{\text{e.g. } n \text{ friends}} \text{ objects}) = \frac{n!}{k!(n-k)!}$$

$\Downarrow$

Def A combination is a way to choose an unordered subset of k objects from a set of n objects.

The number of combinations equals

$$\binom{n}{k} := \frac{n!}{k!(n-k)!}$$

and is called the binomial coefficient "n choose k"

- Ex $\#(\text{ways to choose 3 friends from 7}) = \binom{7}{3} = 35.$