

S1: E4

Recall S1: E2 A permutation of  $n$  objects =  $\forall$  ordered arrangement of them.  
 There are  $1 \cdot 2 \cdot 3 \cdots n = n!$  permutations.

Ex | How many different words can be made by rearranging the letters of the word DOCTOR?

$$\frac{6!}{2} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6}{2} = 360$$

 ↖ all permutations  
 ↘ overcounting correction: ignoring the order of 2 O's

Ex | Same problem for word SUCCESS?

$$\frac{7!}{3! 2!} = 420$$

 ↖ all permutation  
 ↘ ignoring the order of C's  
 ↘ ignoring the order of S's

Ex | In how many ways can Alisa invite 3 from her 7 friends for her party?

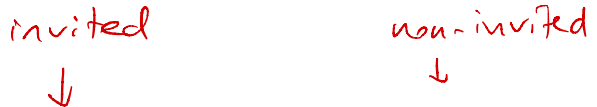
Solution 1

The invitation list = word with 3 letters Y and 4 letter N:

Friend	1	2	3	4	5	6	7
Invited?	N	Y	N	N	Y	N	Y

$$\# \text{ of such words} = \frac{7!}{3! 4!} = 35$$

 ↖ ignoring the order of Y's and N's



Solution 2 :

#(ways to send invitations to 3 friends) =  $7 \cdot 6 \cdot 5$

*invite 1<sup>st</sup> 2<sup>nd</sup> 3<sup>rd</sup>*

Ignore the order of invitations  $\Rightarrow$

$$\frac{7 \cdot 6 \cdot 5}{3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3! \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{7!}{3!4!} = 35.$$

• More generally :

*e.g. k friends*      *e.g. n friends*

#(ways to choose k objects from n objects) =  $\frac{n!}{k!(n-k)!}$

↓

Def A combination is a way to choose an unordered subset of k objects from a set of n objects.

The number of combinations equals

$$\binom{n}{k} := \frac{n!}{k!(n-k)!}$$

and is called the binomial coefficient "n choose k"

• Ex #(ways to choose 3 friends from 7) =  $\binom{7}{3} = 35.$