- Recall binomial coefficient $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$ = # ways to choose an <u>unordered</u> set of k objects from n objects.

- Mind the order (important or not?)

Ex (a) How many ways are there to divide 20 students into a "pro" and "contra" debate teams, 10 students each?

$$\binom{20}{10} = 184,756$$

(b) How many ways are there to divide 20 soccer players into 2 teams, 10 players each?

$$\binom{20}{10} \cdot \frac{1}{2} = 92,378$$

$\uparrow$

ignore the order.

Ex To play a lottery, one picks 5 different #'s (in ∀ order) from 1 to 69, and a "Powerball" number from 1 to 26. If all of these 6 numbers match the winning #'s, one wins the jackpot = $545 Million
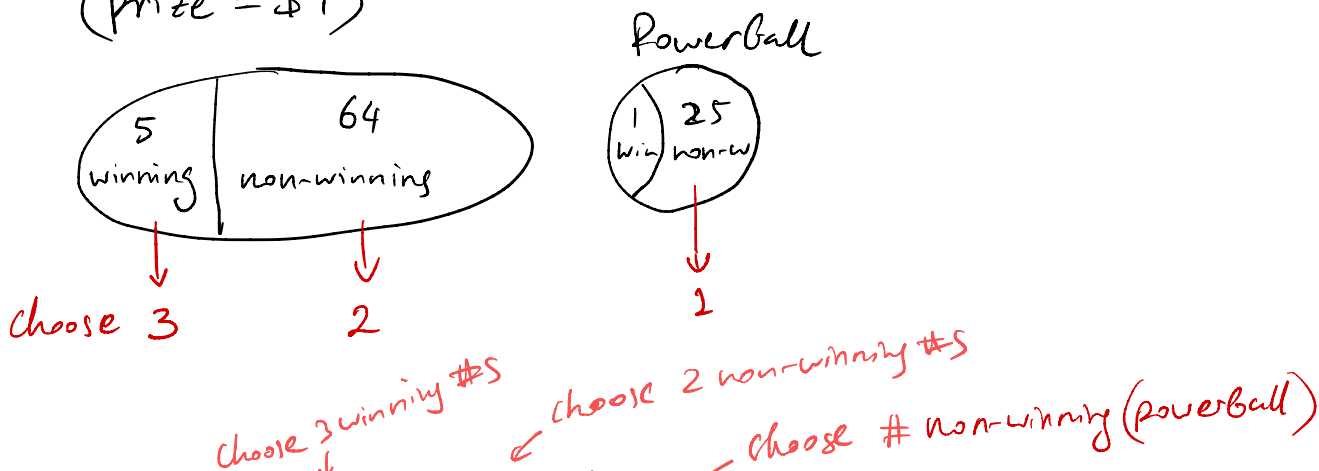
(a) What is prob. to win?

$$\text{\# of ways to play} = \binom{69}{5} \cdot 26 = 292,201,338.$$

Only one way wins $\Rightarrow$ prob $= \dfrac{1}{292,201,338}$.

Note: this prob. $= \frac{1}{50}$ prob (struck by lightning this year).

(b) What is the prob. to guess exactly 3 #'s correctly (and not Powerball) (prize = $7)

Powerball

choose 3   2   1

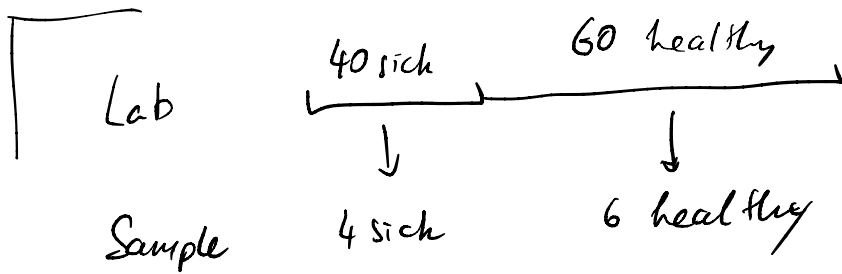choose 3 winning #'s   choose 2 non-winning #'s   choose # non-winning (powerball)

$$\frac{\binom{5}{3}\binom{69-5}{2} \cdot 25}{\binom{69}{5} \cdot 26} = \frac{504,000}{292,201,338} \approx 0.017$$

$\Rightarrow$ fair price to play = $1.7

—2—

Ex | A lab has 100 rats, 40% of which are sick.
A sample of 10 rats is selected for a test.

What is the probability that the sample
is representative, i.e. contains 40% of sick rats?

Lab   40 sick   60 healthy

Sample   4 sick   6 healthy

\# ways to select a sample $= \binom{100}{10}$.

\# ways to select a representative sample $= \binom{40}{4}\binom{60}{6}$.

$$\Rightarrow \text{probability} = \frac{\binom{40}{4}\binom{60}{6}}{\binom{100}{10}} \approx \boxed{0.26}$$

Remark: an <u>approximately</u> representative sample,
e.g. between 39 – 41% sick, has higher probability.

$\rightarrow 1$ as total \# rats $\rightarrow \infty$ (Law of large \#'s)