

SI: E7

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$\dots$$
  
$$(x+y)^5 = (x+y)(x+y)(x+y)(x+y)(x+y)$$

$$= 1 \cdot y^5 + \binom{5}{1} x^1 y^4 + \binom{5}{2} x^2 y^3 + \binom{5}{3} x^3 y^2 + \binom{5}{4} x^4 y^1 + 1 \cdot x^5$$

↑ choose 1 factor for x & the other 4 for y      ... choose 3 factors for x & the other 2 for y

$$= y^5 + 5xy^4 + 10x^2y^3 + 10x^3y^2 + 5x^4y + x^5$$

Generally:

↓

BINOMIAL THEOREM

$\forall x, y \in \mathbb{R}, \forall n \in \mathbb{N}:$

$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$$

Letting  $x=y=1$  in B.T, we get

COROLLARY  $\sum_{k=0}^n \binom{n}{k} = 2^n$

Hard to prove directly

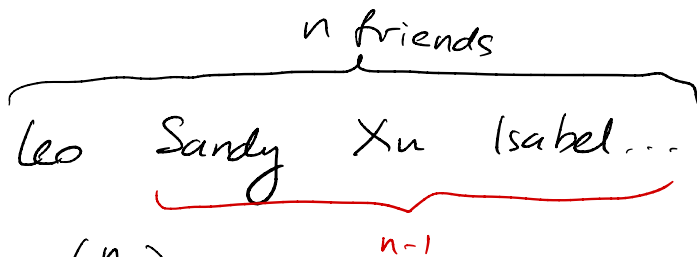
Alternative proof: Both sides = #ways to choose any subset of  $n$  friends for a party

Another identity:

Pascal's Rule  $\forall n, k: \binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$

Prove directly (DIY)

Combinatorial proof:



$$\#(\text{ways to invite } k \text{ friends}) = \binom{n}{k}$$

$$\parallel$$
$$\#(\text{ways to invite } k \text{ friends NOT including leo}) + \#(\dots \text{including leo})$$

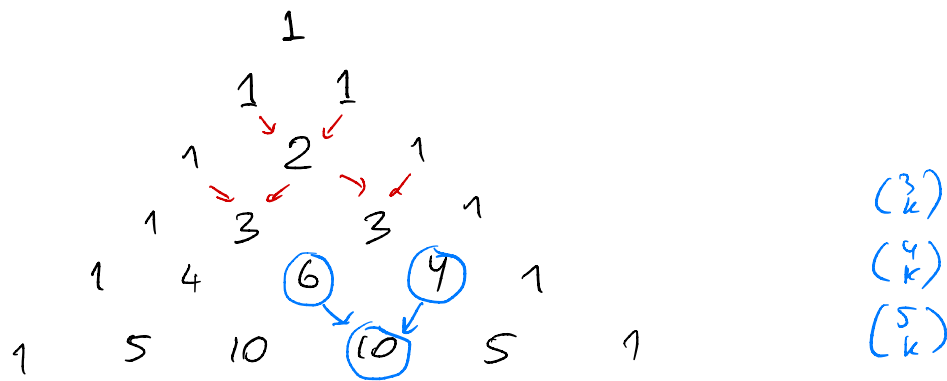
$$= \binom{n-1}{k} + \binom{n-1}{k-1}$$

leo is excluded      leo is already included

write k-1 more

□

P.R yields a recursion to compute binomial coeffs:



$$\text{Thus } (x+y)^5 = y^5 + 5xy^4 + 10x^2y^3 + 10x^3y^2 + 5xy^4 + x^5$$

$$(x+y)^6 = \dots \quad (\text{DIY}).$$