$\boxed{S2:E2}$

OPERATIONS ON EVENTS

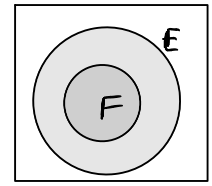
= set operations, interpreted in probability.

Def Consider events $E, F \subset S$ ← sample space

Venn Diagram:

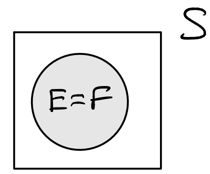
- $\boxed{E \subset F}$ if $\forall s \in E$, $s \in F$. (outcome)

  "E implies F"



- $\boxed{E = F}$ if $E \subset F$ and $F \subset E$

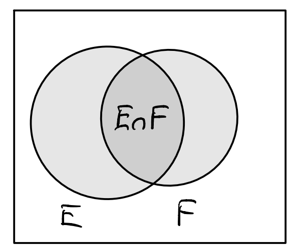
  Either both E, F occur or both don't.



- $\boxed{E \cap F} := \{ s \in S : s \in E \text{ and } s \in F \}$

  ↑ Intersection

  "E AND F occur"



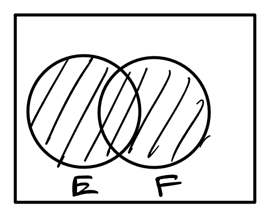
- E, F are mutually exclusive, a.k.a. "disjoint", if

  if $E \cap F = \emptyset$.

- $\boxed{E \cup F} := \{ s \in S : s \in E \text{ or } s \in F \}$

  ↑ Union

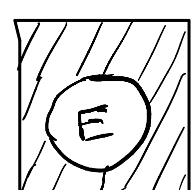
  "E OR F occur" (or both)



- $\boxed{E^c} := \{ s \in S : s \notin E \}$

  ↑ complement

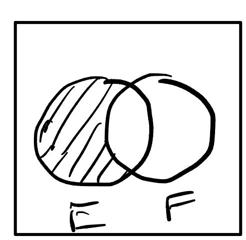
  "E does NOT occur"



- $\boxed{E \setminus F} := E \cap F^c = \{ s \in S : s \in E \text{ and } s \notin F \}$

  ↑ difference

  "E occurs but not F"



Back to examples on p. 1:

1. Flip coin twice. $S = \{HH, HT, TH, TT\}$

$$\left.\begin{array}{l} E = \text{"head once"} \\ F = \text{"tail once"} \end{array}\right\} \Rightarrow \underline{E = F} = \{TH, HT\}$$

2. Record the time of first 911 call $S = [0, 24)$

$$\left.\begin{array}{l} E = \text{"after 2pm"} \\ F = \text{"by 3pm"} \end{array}\right\} \quad \underline{E \cap F} = \text{"between 2-3 pm"} = (2, 3]$$

3. Toss 2 dice. $S = \{(i,j) : i,j = 1, \dots, 6\}$

$$\left.\begin{array}{l} E = \text{"the sum of the dice } \geq 10\text{"} \\ F = \text{"the sum is } \geq 5\text{"} \end{array}\right\} \Rightarrow E \subset F$$

4. Record the sex of children in the family: $S = \{N, B, G, BB, BG, \dots\}$

$E = $ "just two boys" $= \{BB\}$; $F = $ "just two girls" $= \{GG\}$

$E \cap F = \emptyset$, mutually exclusive

$E \cup F = \{BB, GG\} = $ "two children of same gender".

Remark For multiple events, notation:

$$E_1 \cap E_2 \cap \dots \cap E_n = \boxed{\bigcap_{i=1}^{n} E_i} = \text{"all } E_i \text{ occur"}$$

$$E_1 \cup E_2 \cup \dots \cup E_n = \boxed{\bigcup_{i=1}^{n} E_i} = \text{"at least one } E_i \text{ occurs"}$$

Ex A student qualifies for financial aid if s/he passes both English and Finance classes. $Q = E \cap F$

Ara is disqualified = she must have failed either English or Finance or both. $Q^c = E^c \cup F^c$

$$\Downarrow$$

$$(E \cap F)^c = E^c \cup F^c$$

Generally:

THM (De Morgan's laws) $\forall$ events $E, F \in S$:

(a) $(E \cap F)^c = E^c \cup F^c$

(b) $(E \cup F)^c = E^c \cap F^c$

More generally:

$$\left(\bigcap_{i=1}^{n} E_i\right)^c = \bigcup_{i=1}^{n} E_i^c$$

$$\left(\bigcup_{i=1}^{n} E_i\right)^c = \bigcap_{i=1}^{n} E_i^c$$

Ex A coffee maker consists of n components.

CM works $\iff$ all n components work (E_i)

$$\left(\bigcap_{i=1}^{n} E_i\right)^c = \bigcup_{i=1}^{n} E_i$$

CM fails $\iff$ at least one component fails