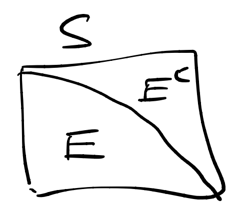
PROPERTIES OF PROBABILITY

Prop. 4.1 (Complement). $\forall$ event E, $P(E^c) = 1 - P(E)$



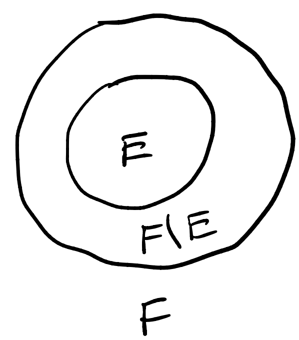
$\ulcorner$ $E \cup E^c = S \Rightarrow$

$P(E) + P(E^c) = P(S)$ (Axiom 1)

$= 1$ (Axiom 2) $\lrcorner$

Cor $P(\emptyset) = P(S^c) \underset{\text{Prop}}{=} 1 - P(S) \underset{\text{Axiom}}{=} 1 - 1 = 0$ $\quad \boxed{P(\emptyset) = 0}$

Prop. 4.2 (Monotonicity). $E \subset F \Rightarrow P(E) \le P(F)$



$\ulcorner$ $F = E \cup (F \setminus E) \Rightarrow$

mutually exclusive (check!)

$P(F) = P(E) + \underbrace{P(F \setminus E)}_{\substack{\vee| \\ 0}}$ (Axiom 3) (Axiom 1)

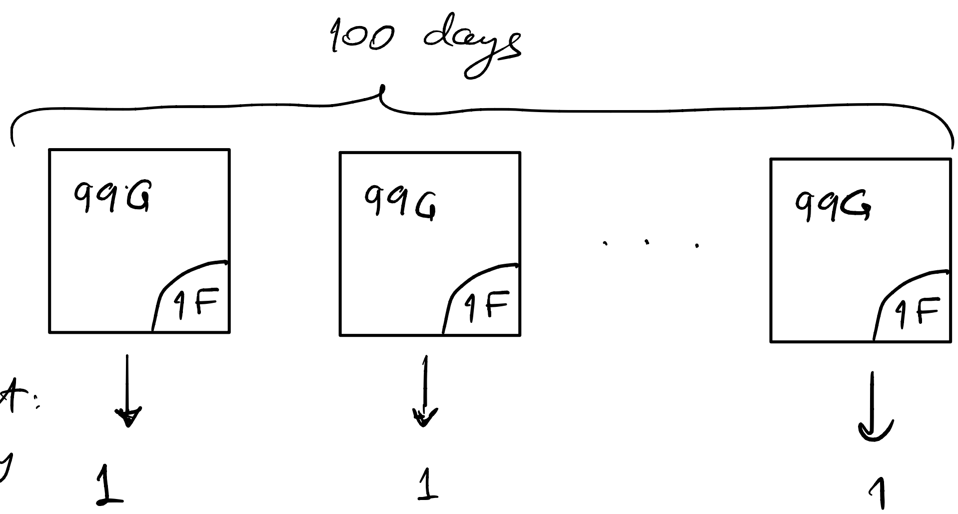
$\ge P(E)$. $\lrcorner$

$\forall$ event $E \subset S \Rightarrow P(E) \le P(S) \overset{\text{Ax. 2}}{\le} 1 \overset{\text{Ax. 1}}{\Longrightarrow}$

Cor $\boxed{0 \le P(E) \le 1}$ $\forall$ event E

Ex

The ministry of propaganda publishes 99 genuine news & 1 fake news story every day. A fact checking agency checks the authenticity of 1 randomly chosen story every day. What is the probability that the ministry's dishonesty gets exposed by 100^{th} day? $=: E$



Experiment: select 1 story every day

$S = \{\text{all ways to select 1 story every day}\}$

$$|S| = \underbrace{100 \cdot 100 \cdots 100}_{100}$$

$E^c =$ "dishonesty is NOT exposed"

$= \{\text{all ways to select 1 genuine story every day}\}$

$$|E| = \underbrace{99 \cdot 99 \cdots 99}_{100}$$

$$\Rightarrow \quad P(E^c) = \frac{|E^c|}{|S|} = \left(\frac{99}{100}\right)^{100} = \left(1 - \frac{1}{100}\right)^{100} \approx \frac{1}{e}.$$

$$\Rightarrow \quad P(E) = 1 - P(E^c) \approx 1 - \frac{1}{e} \approx 0.63$$