"Smokers are more likely to get cancer than non-smokers" is a statement about conditional probabilities.

Study: 360 people:  

<table>
<thead>
<tr>
<th>Cancer</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smoker</td>
<td>8</td>
</tr>
<tr>
<td>No</td>
<td>16</td>
</tr>
</tbody>
</table>

\[
P(\text{Cancer}) = \frac{8+16}{360} \approx 0.07
\]

\[
P(\text{Cancer} \mid \text{Smoke}) = \frac{8}{8+32} = 0.2
\]

\[
P(\text{Cancer} \mid \text{No smoke}) = \frac{16}{16+304} = 0.05
\]

Note calculation above:

"IF", or "GIVEN THAT..."

\[
P(\text{Cancer} \mid \text{Smoke}) = \frac{P(\text{Cancer} \cap \text{Smoke})}{P(\text{Smoke})}
\]

Def Consider events \( E, F \subset \Omega \) with \( P(F) > 0 \).

The conditional probability of \( E \) given \( F \) is

\[
P(E \mid F) = \frac{P(E \cap F)}{P(F)}
\]

we assume \( F \) holds

Remark: \( F \) becomes a new probability space.
Consider families with 2 children

(a) If the older child is a girl, what is the probability that both children are girls?

\[ S = \{ GG, GB, BB, BB \} \quad \text{(older first; all equally likely)} \]

\[ F = \{ GG, GB \}, \quad E = \{ GG \} \]

\[ P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/4}{2/4} = \frac{1}{2} \]

(b) If at least one child is a girl, what is the probability that both children are girls?

\[ F = \{ GG, GB, BG \}, \quad E = \{ GG \} \]

\[ P(E|F) = \frac{1}{3} = \frac{1}{3} \quad \text{Surprising} \]
Ex: 75% people live at least 70 years.  
63% people live at least 80 years.  
For a 70 y.o. person, what is the probability to live at least 10 more years?

\[ P(E) = 0.63, \quad P(F) = 0.75 \]

\[ P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{P(E)}{P(F)} = \frac{0.63}{0.75} = 0.84 \]

Ex: On a given day, a typical person opens Netflix with prob. 0.2; then either rents a movie with prob. 0.15 or closes Netflix with prob. 0.85. What is the probability that a person rents a Netflix movie on a given day?

\[ P(\text{EnF}) = P(F) \cdot P(\text{E} \mid F) = 0.2 \times 0.15 = 0.03 \]

a useful “multiplication rule”