S3: E1

CONDITIONAL PROBABILITY

- "Smokers are more likely to get cancer than non-smokers"

is a statement about conditional probabilities.

Study: 360 people:

	Cancer	No
Smoker	8	32
No	16	304

$$P(\text{Cancer}) = \frac{8+16}{360} \approx 0.07$$

$$P(\text{Cancer} \mid \text{Smoke}) = \frac{8}{8+32} = 0.2$$

$$P(\text{Cancer} \mid \text{No smoke}) = \frac{16}{16+304} = 0.05$$

4 x higher

- Note calculation above:

"IF", or "GIVEN THAT..."

$$P(\text{Cancer} \mid \text{Smoke}) = \frac{P(\text{Cancer} \cap \text{Smoke})}{P(\text{Smoke})}.$$

Def Consider events $E, F \subset S$ with $P(F) > 0$.
The conditional probability of E given F is

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$

we assume F holds

Remark F becomes a new probability space.

Ex Consider families with 2 children

(a) If $\overbrace{\text{the older child is a girl}}^{F}$, what is the probability that $\underbrace{\text{both children are girls}}_{E}$?

$S = \{GG, GB, BG, BB\}$ (older first; all equally likely)

$F = \{GG, GB\}$, $E = \{GG\}$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/4}{2/4} = \frac{1}{2}$$

(b) If $\overbrace{\text{at least one child is a girl}}^{F}$, what is the probability that $\underbrace{\text{both children are girls}}_{E}$?

$F = \{GG, GB, BG\}$, $E = \{GG\}$

$$P(E|F) = \frac{1/4}{3/4} = \frac{1}{3}.$$

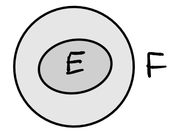
Surprising

Ex 75% people live at least 70 years $\leftarrow F$

63% people live at least 80 years. $\leftarrow E$

For a 70 y.o. person, what is the probability to live at least 10 more years?

$P(E) = 0.63$, $P(F) = 0.75$



$$P(E|F) = \frac{P(E\cap F)}{P(F)} \underset{E\subset F}{=} \frac{P(E)}{P(F)} = \frac{0.63}{0.75} = 0.84$$

Ex On a given day, a typical person opens Netflix with prob. 0.2; $\leftarrow F$
then either rents a movie with prob. 0.15
or closes Netflix with prob. 0.85.
What is the probability that a person rents a Netflix movie on a given day?

rents a movie, Netflix

$$P(E\cap F) = P(F)\cdot P(E|F) = 0.2\times 0.15 = 0.03$$

a useful "multiplication rule"