Conditional Probability : $P(E|F) = \dfrac{P(E\cap F)}{P(F)}$.

$\Rightarrow$ "Chain rule" $\boxed{P(E\cap F) = P(E|F)\cdot P(F)}$

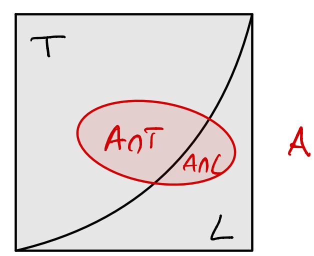
Ex 70% of Southwest flights depart on time

80% of flights that $\overbrace{\text{depart on time}}^{T}$, arrive on time.

90% of flights that $\overbrace{\text{depart late}}^{L}$ arrive late.

What % of flights $\overbrace{\text{arrive on time}}^{A}$?

Choose a flight uniformly at random.



$A = \underbrace{(A\cap T)}\cup\underbrace{(A\cap L)}$

mutually exclusive

$\Rightarrow P(A) = P(A\cap T) + P(A\cap L)$

$= \underbrace{P(A|T)}_{=\,0.8}\cdot\underbrace{P(T)}_{=\,0.7} + \underbrace{P(A|L)}_{=\,1-0.9}\cdot\underbrace{P(L)}_{=\,1-0.7}$ $\leftarrow$ def. of conditional probability

$= 0.59$. Ans = 59%

More generally ?

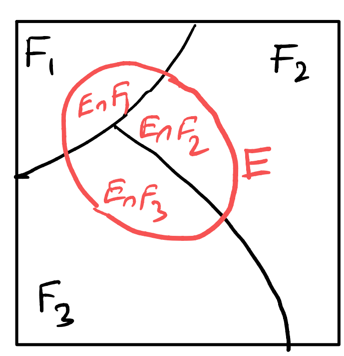
Thm (Law of Total Probability) Assume that

$$S = \bigcup_{i=1}^{N} F_i$$

where $F_i \subset S$ are mutually exclusive events.

Then $\forall$ event $E \subset S$,

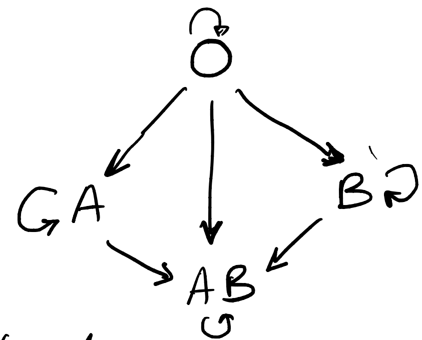
$$P(E) = \sum_{i=1}^{N} P(E|F_i)\,P(F_i)$$



- LTP allows to compute probabilities by "conditioning" on F_i, use extra information (e.g. whether departure is late or not).

Ex (Blood transfusion) Each person's blood is one of the following 4 types: O, A, B, AB.

The chart of donor-recipient compatibility:



34% of people have O type, 38% A type, 20% B type, 8% AB type.

Find the probability that a randomly matched $\underbrace{\text{donor-recipient pair is OK}}_{E}$?

Condition on the donor type. LTP:

$$P(E) = \underbrace{P(E|O)}_{=1} P(O) + \underbrace{P(E|A)}_{=P(A)+P(AB)} P(A) + \underbrace{P(E|B)}_{=P(B)+P(AB)} P(B) + \underbrace{P(E|AB)}_{=P(AB)} P(AB)$$

(donor O can give blood to $\forall$ recipient)

(donor A gives blood to A & AB)

$$= 1 \cdot 0.34 + (0.38 + 0.08)\, 0.38 + (0.2 + 0.08)\, 0.2 + 0.08 \cdot 0.08$$

$$\simeq \boxed{0.58}$$