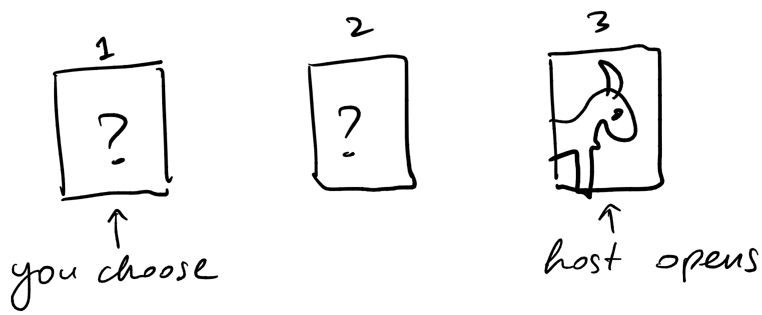
S3 : E4

Ex (Monty Hall Problem)

- In a game show, you are given the choice of 3 doors. Behind one door is a car; behind the other two, goats.
- The host knows what is behind each door; you don't.
- You choose a door, say No. 1.
- The host opens another door, say No. 3, and you see a goat.
- The host offers you to switch to Door No. 2. Should you switch?



- C_i = "car behind door i"
- Assumptions on the host behaviour: wants to keep the show going as long as possible

  Thus, after you pick door No. 1:

(a) $C_1 \Rightarrow$ host opens door 2 or 3 at random (50-50)

(b) $C_2 \Rightarrow$ host opens door 3

(c) $C_3 \Rightarrow$ host opens door 2.

- D_3 = "host opens door 3"

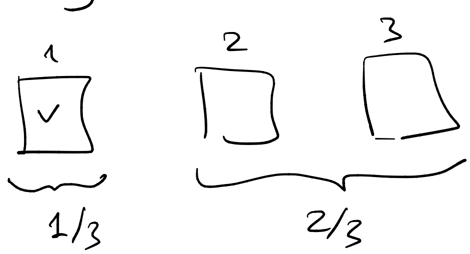
$$P(C_2|D_3) \overset{\text{Bayes}}{=} \frac{P(D_3|C_2)P(C_2)}{P(D_3)} \overset{\text{LTP}}{=} \frac{\overbrace{P(D_3|C_2)}^{=1 \text{ by } (b)}\ \overbrace{P(C_2)}^{=1/3 \text{ (initially, car can be in } \forall\, 3 \text{ doors) equally likely}}}{\underbrace{P(D_3|C_1)}_{=\frac{1}{2} \text{ by } (a)}\underbrace{P(C_1)}_{=\frac{1}{3}} + \underbrace{P(D_3|C_2)}_{=1 \text{ by } (b)}\underbrace{P(C_2)}_{=\frac{1}{3}} + \underbrace{P(D_3|C_3)}_{=0 \text{ by } (c)}\underbrace{P(C_3)}_{=\frac{1}{3}}} = \frac{2}{3}$$

vs.

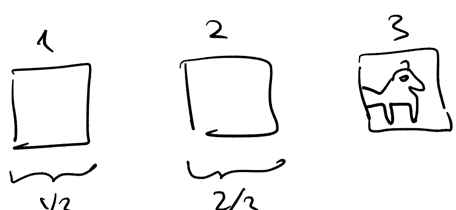
$$P(C_1|D_3) = \frac{\overbrace{P(D_3|C_1)}^{=1/2\,(a)}\ \overbrace{P(C_1)}^{=1/3}}{P(D_3)} = \frac{1}{3}$$

Hence, you should always switch, thereby doubling your chances to win.

- Paradox.
- Explanation
  - When you make your first choice (door 1), there is $\frac{2}{3}$ probability that the car is behind the 2 unchosen doors:



  - When the host reveals the goat behind one of the unchosen doors (3), prob. $2/3$ now rests on the other unchosen door (2):



- Equivalent to the "Three prisoners problem"