S3:E6

Ex (Finding your Birthmate).

How many strangers do you need to ask to have a 50-50 chance to find someone with the same birthday as yours?

n ... E

$E = \bigcup_{i=1}^{n} E_i$ where E_i = "i'th stranger is your Birthmate"

E_i are independent, $P(E_i) = \frac{1}{d}$, where $d = 365$.

$$P(E^c) = P\left(\bigcap_{i=1}^{n} E_i^c\right) \qquad \text{(de Morgan)}$$

$$= \prod_{i=1}^{n} \underbrace{P(E_i^c)}_{= 1-\frac{1}{d}} \qquad \left(\text{stability of independence} \Rightarrow E_i^c \text{ are indep.}\right)$$

$$= \left(1-\frac{1}{d}\right)^n = \Bigg[\underbrace{\left(1-\frac{1}{d}\right)^d}_{\approx 1/e}\Bigg]^{\frac{n}{d}} \qquad \left(\text{since } \lim_{d\to\infty}\left(1-\frac{1}{d}\right)^d = \frac{1}{e}\right)$$

$$\approx e^{-n/d} =: \frac{1}{2} \qquad \text{(50 - 50 chance)}$$

Solve for $n \Rightarrow$

$$n = \underbrace{d}_{=365} \ln 2 \approx 253 \quad \leftarrow \text{Answer.}$$

Remark: the approximation is very accurate: $n = 253$ gives $0.5005 - 0.4995$ chance.