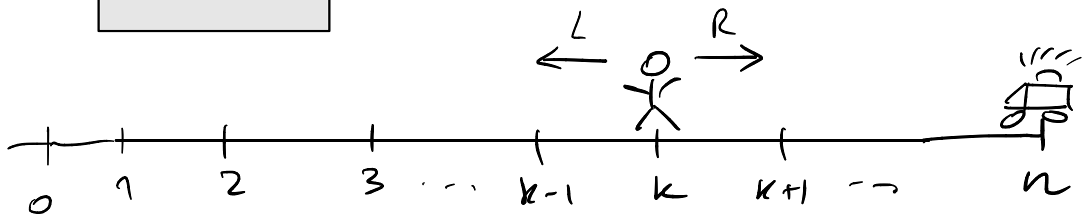
• Simple random walk:



A particle is placed at k.
Each second, moves 1 step to the right or to the left independently with prob $1/2$ each.

• <u>Ex. 1</u> (Gambler's Ruin)

What is probability of reaching n before reaching 0?
$= E_k$

Condition on 1^{st} step, L or R:

$$P(E_k) = P(E_k \mid L)\, P(L) + P(E_k \mid R)\, P(R)$$

$$= P(E_{k-1}) \cdot \frac{1}{2} + P(E_{k+1}) \cdot \frac{1}{2}$$

walk "resets" at $k-1$ walk "resets" at $k+1$

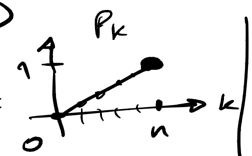
Denoting $p_k = P(E_k)$, we obtain

$$\begin{cases} p_k = \frac{1}{2}(p_{k-1} + p_{k+1}), & k = 1, \dots, n-1 \\ p_0 = 0; \quad p_n = 1 \end{cases}$$

$n+1$ linear equations in $n+1$ unknowns. Solve →

$$\boxed{p_k = \frac{k}{n}}$$

(obviously, it is a solution:

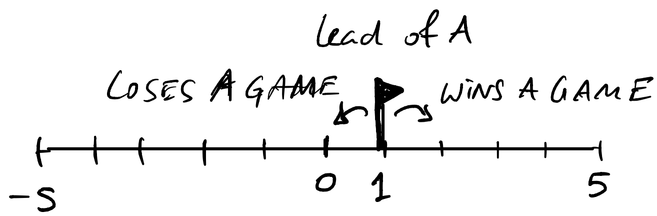


• <u>Remark</u>: <u>Finance</u>: 0 = Bankrupcy, n = payoffs
k = initial capital

Ex2. Teams A and B, who have equal strength, play against each other continually.

1 point is awarded to the winner of each game.
The first team that leads by 5 points wins the tournament
Team A currently leads by 1 pt.
What is the prob. that team A wins the tournament?

- Assumptions: outcomes of the games are independent, equally likely win/lose



$\Leftrightarrow$

- The "lead of A" is doing a simple random walk, starting at 1

A wins tournament $\Leftrightarrow$ starting at 1, the lead reaches 5 before -5.

- By shifting, $+5$ $\Leftrightarrow$ starting at 6, the lead reaches 10 before 0.

From the previous problem, this probability $= \frac{6}{10} = 0.6$

Read more on random walk