RANDOM VARIABLES

Ex (a) Flip 3 coins at random. Observe # heads:

$$S = \{TTT, TTH, THT, THH, HTT, HTH, HHT, HHH\}$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$0 \quad 1 \quad 1 \quad 2 \quad 1 \quad 2 \quad 2 \quad 3$$

$\Rightarrow$ we assign a number $X(s)$ to each outcome $s \in S$

Any such assignment is called a <u>random variable X</u>

Def A random variable is a real-valued function on the sample space S:
$$X : S \to \mathbb{R}$$

(b) X = the age of a randomly chosen person in this class

$$S = \{\text{Xavier}, \text{Tycho}, \text{Chi}, \text{Sara}, \ldots\}$$
$$X: \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$20 \quad 19 \quad 21 \quad 24$$

(c) X = # enrolled students at UCI next year. ($S = ?$)

(d) The number of spam calls in 2021 [$S = ?$ The role of S will be downplayed]

(e) The delay of a future flight

(f) Molecule's velocity at $\forall$ moment

(g) The age of a fossil

(h) An (imperfect) measurement of the distance to some star.

DISCRETE R.V'S

Def A r.v. X is <u>discrete</u> if $X: S \to \mathbb{R}$ takes on a finite or countable # of values $x_1, x_2, \ldots$

Ex (a)–(d) are discrete; (e)–(h) are continuous.

- The <u>distribution</u> of X is the knowledge of what values X takes and with what probabilities. Can be expressed in 2 ways:

① **Def** Let X be a discrete r.v.
The <u>probability mass function</u> (pmf) of X is defined as

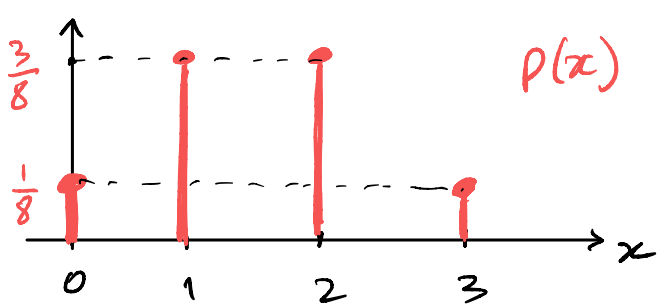
$$p(x) = P\{X = x\}, \qquad x \in \mathbb{R}$$

formally, $P\{s \in S : X(s) = x\}$

Ex X = # heads in 3 coin flips (see p.1)

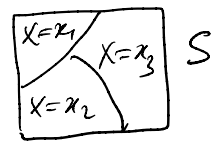
$$p(0) = \frac{1}{8}, \quad p(1) = \frac{3}{8}, \quad p(2) = \frac{3}{8}, \quad p(3) = \frac{1}{8}$$

Everywhere else, $p(x) = 0$



Prop (total sum) If X takes on values $x_1, x_2, \ldots$, then

$$\boxed{\sum_i \underbrace{p(x_i)} = 1.}$$



$P\{X = x_i\}$ Events $F_i = \{X = x_i\}$ form a partition of S

$\Rightarrow$ By additivity axiom, $\sum_i P(F_i) = P(S) = 1.$

Ex above: $\frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1$

Ex $X = \#$ heads in n coin flips.

$S = \{$all 2^n strings of length n with letters $H, T\}$,

uniform probability $\frac{1}{2^n}$ (by independence)

$$p(k) = P\{X = k\} = P\{k \text{ heads in } n \text{ flips}\}$$

$\underbrace{HTTHHTTH}_{n}$

$$= \frac{\#(\text{words with } k \text{ letters } H)}{|S|}$$

$$= 2^{-n}\binom{n}{k}.$$