

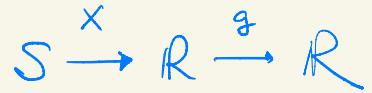
Expectation of a function of a r.v.

- Recall: If r.v. X takes values x_i with prob. p_i , expectation ("mean"):

$$E[X] = \sum_i x_i p_i$$

- If $g: \mathbb{R} \rightarrow \mathbb{R}$ is a function, $g(x) = g \circ X$ is a r.v.

that takes values $g(x_i)$ with prob. $p_i \Rightarrow$



$$E[g(X)] = \sum_i g(x_i) p_i, \text{ e.g. } E[X^2] = \sum_i x_i^2 p_i$$

VARIANCE

- How much X deviates from $\mu = E[X]$?
- First attempt: $E|X-\mu|$. Abs. value is inconvenient.
The square is better:

Def The variance of a r.v. X with expectation ("mean") $\mu = E[X]$ is

$$\text{Var}(X) := E[(X-\mu)^2]$$

Standard deviation is
 $\sigma(X) = \sqrt{\text{Var}(X)}$

Expand the square:

$$\begin{aligned} \text{Var}(X) &= E[X^2 - 2X\mu + \mu^2] \\ &= E[X^2] - 2\mu E[X] + \mu^2 \quad (\text{linearity}) \\ &= E[X^2] - \mu^2. \end{aligned}$$

Prop. $\text{Var}(X) = E(X^2) - (E[X])^2$

↖ ↗
not the same #'s

Ex Compare 2 investment options:

Stock A: 3% return with prob 0.8; 2% loss with prob. 0.2

Stock B: 5% return with prob. 0.6; 2% loss with prob. 0.4.

Expected profit: $E(X_A) = 3 \cdot 0.8 - 2 \cdot 0.2 = 2\%$

$E(X_B) = 5 \cdot 0.6 - 2 \cdot 0.4 = 2.2\%$ ← a little better.

however, compare the risks:

$$E(X_A^2) = 3^2 \cdot 0.8 + 2^2 \cdot 0.2 = 8$$

$$E(X_B^2) = 5^2 \cdot 0.6 + 2^2 \cdot 0.4 = 16.6$$

Prop $\Rightarrow \text{Var}(X_A) = 8 - 2^2 = 4 ; \quad \text{Var}(X_B) = 16.6 - (2.2)^2 = 11.8$

St. deviation:

$$\sigma(X_A) = \sqrt{4} = 2 ; \quad \sigma(X_B) = \sqrt{11.8} \approx 3.4$$

\Rightarrow typical return for Stock A: $2 \pm 2\%$

Stock B: $2.2 \pm 3.4\%$

\Rightarrow Stock A has almost same return with less risk