Exercise 1  Classification of differential equations

For the following differential equations, find out (a) its order, (b) whether it’s linear or nonlinear and (c) if it’s nonlinear, what the terms are that show it’s nonlinear.

1. \( y''' + 2y' + y + e^x = 0 \).
2. \( 2 \frac{dx}{dy} + xy = 0 \).
3. \( yy'' + ty' + y = 0 \).
4. \( y'' + e^y + y + x = 0 \).

Exercise 2  Direction Fields

(a) For the equation \( y' = y - 2 \) for a function \( y(x) \), draw its direction field with \( x \) on the horizontal axis. (Hint: you need to divide the plane into at least three regions.) When \( x \) approaches \( +\infty \), what does \( y \) approach?

(b) For the equation \( y' = -y - 2 \) for a function \( y(x) \), draw its direction field with \( x \) on the horizontal axis. When \( x \) approaches \( +\infty \), what does \( y \) approach?

(c) Find the equilibrium solution(s) of the equation \( y' = y(y - 3) \).

Exercise 3  Math Modeling

Consider the equation

\[
\frac{dp}{dt} = 0.5p - 450. \tag{1}
\]

This equation has been used as a model of a field mouse population.

(a) Find the time at which the population becomes extinct if \( p(0) = 850 \).

(b) Find the time of extinction if \( p(0) = p_0 \), where \( p_0 \) is a number between 0 and 900.

(c) Find the initial population \( p_0 \) if the population is to become extinct when \( t = 12 \).

Exercise 4  Verify solutions
(a) For $y''' + 4y'' + 3y = t$, verify that $y_1(t) = t/3$ and $y_2(t) = e^{-t} + t/3$ are solutions.
(b) For $ty' - y = t^2$, verify that $y(t) = 3t + t^2$ is a solution.

Exercise 5  *Separable Equations*

(a) Solve equation

$$\frac{dy}{dx} = \frac{\sqrt{y^2 - a^2}}{a},$$

with the initial condition $y(0) = a$.

Your answer should be a function of $x$ with the parameter $a$ still present.

(b) Solve the initial value problem

$$\frac{dr}{d\theta} = \frac{r^2}{\theta}, \quad r(1) = 2.$$

(c) Solve the initial value problem

$$y' = \frac{3x^2}{3y^2 - 4}, \quad y(1) = 0.$$

(d) Solve the initial value problem

$$y' = \frac{e^{-x} + e^x}{3 + 4y}, \quad y(0) = 1.$$