MATH 120A MIDTERM EXAM

FALL 2014

Student name:

Student ID number:

Instructions

- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- The time remaining will be written on the board periodically.
- When time is called, you must stop working immediately.
- If you want to leave your seat for any reason before time is called, raise your hand and remain seated until acknowledged.

1	/ 10
2	/ 4
3	/ 4
4	/ 4
5	/ 4
6	/ 4
Total	/ 30

Problem 1 (10 points). Mark each statement 'T' for true or 'F' for false. You do NOT need to justify your answers.

T F The set of all 3×3 matrices with entries in the set $\{-1, 0, 1\}$ is closed under matrix multiplication.

T F Addition is a binary operation on the set of all nonzero integers.

T F A binary operation on a set S is a function from S to $S \times S$.

T F If two groups are isomorphic, then their orders (cardinalities) must be the same.

T F The groups $(2\mathbb{Z}, +)$ and $(3\mathbb{Z}, +)$ are isomorphic, where $n\mathbb{Z} = \{nk : k \in \mathbb{Z}\}$.

T F Every group has only finitely many subgroups.

T F Every finite cyclic group is isomorphic to $(\mathbb{Z}_n, +_n)$ for some n.

T F The dihedral group D_n (the group of symmetries of a regular n-gon) has order $n!$.

T F The group of nonzero real numbers under multiplication has a subgroup that is isomorphic to $(\mathbb{Z}, +)$.

T F Every cyclic group is abelian.

Problem 2 (4 points).

(a) Let $(S, *)$ be a binary structure. Define what it means for the operation $*$ to be commutative.

(b) Define precisely what it means for two groups $(G, *)$ and $(G', *')$ to be isomorphic. (If you use the word "isomorphism" in your definition, define that word also.)

Problem 3 (4 points). Prove or disprove the following statement: There a subgroup of the dihedral group D_5 that is isomorphic to $(\mathbb{Z}_2, +_2)$.

Problem 4 (4 points). Let $(G, +)$ be an abelian group (possibly infinite) and let S be the subset of G consisting of all elements of finite order:

$$S = \{a \in G : na = 0 \text{ for some positive integer } n\}.$$

(Note the use of additive notation.) Prove that S is a subgroup of $(G, +)$.

Problem 5 (4 points). By drawing a table, define a binary operation $*$ on the two-element set $\{0, 1\}$ such that $*$ is associative, but the structure $(\{0, 1\}, *)$ is not a group. Justify your answer.

Problem 6 (4 points). Draw the subgroup diagram for $\mathbb{Z}_8$. You do NOT need to prove that your diagram is complete, but it must be complete in order to get full credit.