MATH 120A MIDTERM EXAM

FALL 2014

Student name:

Student ID number:

Instructions

- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- The time remaining will be written on the board periodically.
- When time is called, you must stop working immediately.
- If you want to leave your seat for any reason before time is called, raise your hand and remain seated until acknowledged.

1	/ 10
2	/ 4
3	/ 4
4	/ 4
5	/ 4
6	/ 4
Total	/ 30

Problem 1 (10 points). Mark each statement 'T' for true or 'F' for false. You do NOT need to justify your answers.

- T F If G is an infinite group and $a \in G$ is not the identity, then the order of a must be infinite.
- T F There is only one binary operation that can be defined on the set $\{0\}$.
- T F Any two groups of order 3 are isomorphic.
- T F Every finite group G has an element a such that the order of a is equal to the order of G.
- T F The only automorphism of $(\mathbb{Z}_3, +_3)$ is the identity. (An *automorphism* of a group is an isomorphism of that group with itself.)

- T F Every finite group has only finitely many subgroups.
- T F There is a surjection from $\mathbb{Q}$ onto $\mathbb{R}$.
- T F Any two abelian groups of order 4 are isomorphic.
- T F The dihedral group D_5 has an element of order 3.
- T F There is a binary operation $*$ on the set $\{3, 4, 5\}$ such that $(\{3, 4, 5\}, *)$ is a group.

Problem 2 (4 points).

(a) Let $(S, *)$ be a binary structure. Define what it means for the operation $*$ to be associative.

(b) Let A be a set. How is the symmetric group S_A defined? Be sure to say what its elements are *and* what the operation is.

Problem 3 (4 points). Prove or disprove the following statement: There a subgroup of $(\mathbb{Z}, +)$ that is isomorphic to $(\mathbb{Z}_6, +_6)$.

Problem 4 (4 points). Let $(G, *)$ and $(G', *')$ be groups and let ϕ be a homomorphism from $(G, *)$ to $(G', *')$, meaning that $\phi(x * y) = \phi(x) *' \phi(y)$ for all $x, y \in G$. Let S be the range of ϕ:

$$S = \{x' \in G' : x' = \phi(x) \text{ for some } x \in G\}.$$

Prove that S is a subgroup of G'.

Problem 5 (4 points). By drawing a table, define a binary operation $*$ on the two-element set $\{0, 1\}$ that is not associative. Explain why your operation is not associative.

Problem 6 (4 points). Draw the subgroup diagram for $\mathbb{Z}_{14}$. You do NOT need to prove that your diagram is complete, but it must be complete in order to get full credit.