MATH 120A SAMPLE FINAL EXAM

FALL 2014

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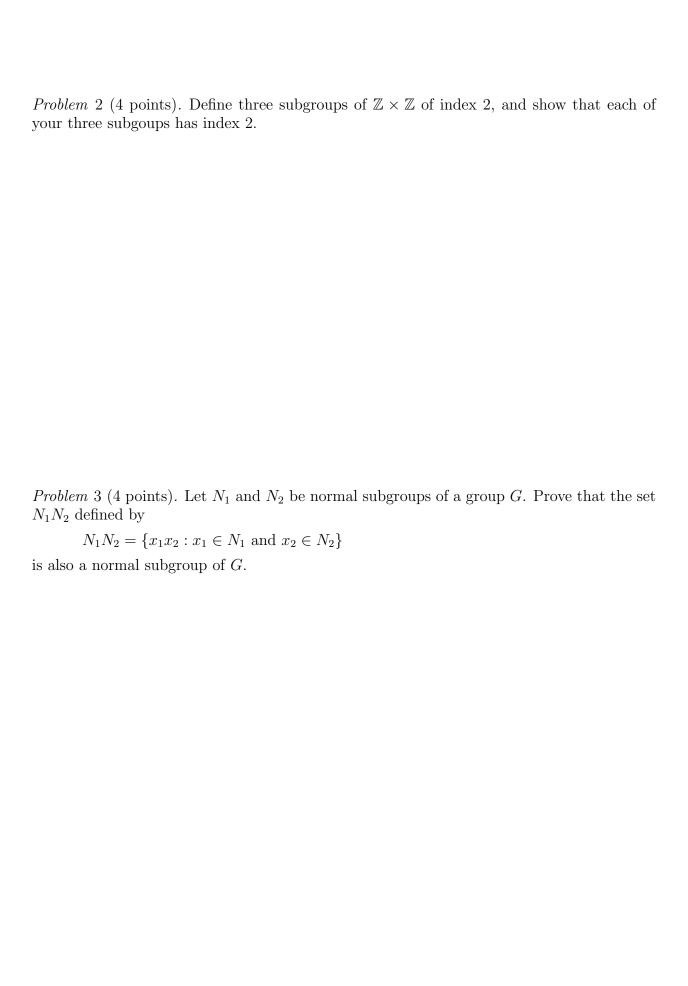
Instructions

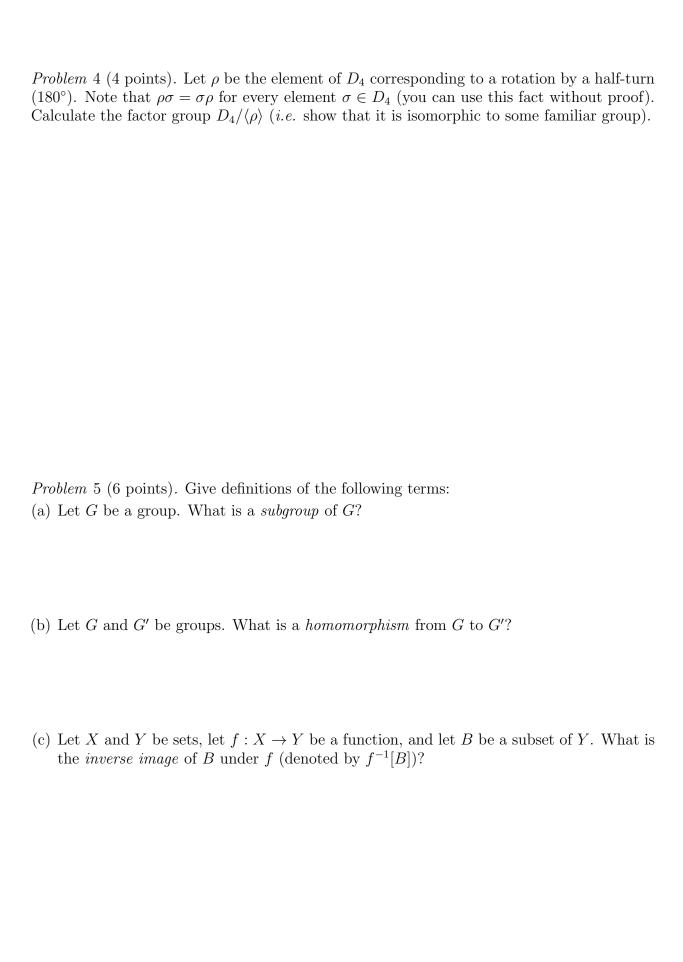
- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- The time remaining will be written on the board periodically.
- When time is called, you must stop working immediately.
- If you want to leave your seat for any reason before time is called, raise your hand and remain seated until acknowledged.

1	/	/	16
2	,	/	4
3	,	/	4
4	,	/	4
5	,	/	6
6	,	/	4
7	,		4
8	/	— /	4
Total	/		46
10001	/		10

Problem 1 (16 points). Mark each statement 'T' for true (meaning always true) or 'F' for false (meaning sometimes false). You do NOT need to justify your answers.

- T F Every subgroup of a cyclic group is cyclic.
- T F There is a bijection from \mathbb{Q} to \mathbb{R} .
- T F Every finite group is isomorphic to a subgroup of S_n for some n.
- T F If $\sigma, \tau \in S_n$ are disjoint cycles, then $\sigma \tau = \tau \sigma$.
- T F If the two permutations $\sigma, \tau \in S_n$ are conjugate to one another, then σ and τ have the same orbits.
- T F Let G be a group and let H be a normal subgroup of G. If G/H and H are cyclic, then G is cyclic.
- T F If G is a group and H is a normal subgroup of G, then H is the kernel of some homorphism $\phi: G \to G'$ for some group G'.
- T F $\mathbb{Z}_3 \times \mathbb{Z}_3$ is isomorphic to \mathbb{Z}_9 .
- T F Let G_1 , G_2 , G_1' , and G_2' be groups. If G_1 is isomorphic to G_1' and G_2 is isomorphic to G_2' , then $G_1 \times G_2$ is isomorphic to $G_1' \times G_2'$.
- T F Let G be a group and let H be a subgroup of G. If ah = ha for all $h \in H$ and $a \in G$, then H is a normal subgroup of G.
- T F The direct product $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic if and only if m and n are relatively prime.
- T F There is a bijection from \mathbb{Z} to \mathbb{Q} .
- T F For all $m, n \in \mathbb{Z}^+$ the direct product of symmetric groups $S_m \times S_n$ is isomorphic to S_{m+n} .
- T F Every element of the symmetric group S_n is a product of disjoint cycles.
- T F If G is a group and n divides the order of G, then G has an element of order n.
- T F Every group of prime order is cyclic.





Problem 6 (4 points). In the symmetric group S_4 :

- How many elements have order 1?
- How many elements have order 2?
- How many elements have order 3?
- How many elements have order 4?
- How many elements have order 5?

Problem 7 (4 points). List, up to isomorphism, all abelian groups of order 225. Which group in your list is isomorphic to $\mathbb{Z}_{15} \times \mathbb{Z}_{15}$? Why?

Problem 8 (4 points). Let G be a group and suppose that $a \in G$ has order 2, but no other element of G has order 2. Prove that ax = xa for all $x \in G$.