MATH 120A SAMPLE MIDTERM EXAM

FALL 2014

Student name:

Student ID number:

Instructions

- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- The time remaining will be written on the board periodically.
- When time is called, you must stop working immediately.
- If you want to leave your seat for any reason before time is called, raise your hand and remain seated until acknowledged.

1	/ 10
2	/ 4
3	/ 4
4	/ 4
5	/ 4
6	/ 4
Total	/ 30

Problem 1 (10 points). Mark each statement 'T' for true or 'F' for false. You do NOT need to justify your answers.

T F Any two groups of order 4 are isomorphic.
T F For every $n \in \mathbb{Z}^+$ there is an element of order n in the group $(\mathbb{Z}, +)$.
T F Multiplication is an associative operation on the set of all 2×2 real matrices.
T F The group $(\mathbb{Z}, +)$ is isomorphic to one of its proper subgroups.
T F The relation $\{(a, x), (a, y)\}$ is a function from the set $\{a, b\}$ to the set $\{x, y, z\}$.

T F Multiplication (meaning composition) is a commutative operation on the set of all permutations of $\{1, 2, 3\}$.
T F Up to isomorphism, there is only one infinite abelian group.
T F If G and H are isomorphic groups and every element of G has order 2, then every element of H must have order 2 also.
T F Every abelian group is cyclic.
T F The group $(\mathbb{Z}_7, +_7)$ has an element of order 6.

Problem 2 (4 points). Let G be a group and let $a \in G$.
(a) Define the *order* of a.

(b) Define what it means for a to be a *generator* of G.

Problem 3 (4 points). Prove or disprove the following statement: The group $(\mathbb{Z}_6, +_6)$ has a subgroup H that is isomorphic to $(\mathbb{Z}_4, +_4)$.

Problem 4 (4 points). Let G be a group and let S be a subset of G. Define another subset C of G by

$$C = \{g \in G : sg = gs \text{ for all } s \in S\}.$$

Prove that C is a subgroup of G.

Problem 5 (4 points). By drawing a table, define a binary operation $*$ on the two-element set $\{a, b\}$ such that the structure $(\{a, b\}, *)$ has an identity element, but is not a group. Explain why your structure has the properties you claim.

Problem 6 (4 points). Up to isomorphism, there are exactly two groups of order 4, namely $\mathbb{Z}_4$ and another group V whose elements we will call e, a, b, and c where e is the identity. Draw a subgroup diagram for $\mathbb{Z}_4$ and another subgroup diagram for V.