MATH 120A MIDTERM EXAM
(WHITE PAPER)

WINTER 2015

Student name:

Student ID number:

Instructions

- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- When time is called, you must stop working immediately, close your exam, and remain seated until your exam is collected.
- If you want to leave your seat for any reason before time is called, raise your hand and remain seated until acknowledged.

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Problem 1 (10 points). Mark each statement 'T' for true (meaning always true) or 'F' for false (meaning sometimes false). You do NOT need to justify your answers to this problem.

T F If every element of the group G_1 is its own inverse, and G_2 is isomorphic to G_1, then every element of G_2 is its own inverse.

T F The set of all positive rational numbers forms a group under multiplication.

T F The set of all 2×2 real matrices of positive determinant is closed under matrix multiplication.

T F The binary structure $(S, \max)$ has an identity element, where $S = \{x \in \mathbb{R} : 0 < x < 1\}$.

T F The group $(\mathbb{R}^*, \cdot)$ of nonzero real numbers under multiplication is isomorphic to the group $(\mathbb{R}^+, \cdot)$ of positive real numbers under multiplication.

T F A binary operation on a set S is a function from $S \times S$ to $S \times S$.

T F There is a bijection from $\mathbb{N}$ to $\mathbb{Q}$.

T F If H is a proper subgroup of G, then $|H| < |G|$.

T F Some element of the group $(\mathbb{Z}_{12}, +_{12})$ has order 4.

T F Every abelian group is cyclic.

Problem 2 (4 points). Define the italicized terms:

(a) What does it mean for a binary operation $*$ on a set S to be *commutative*?

(b) What does it mean for a binary operation $*$ on a set S to be *associative*?

Problem 3 (4 points). Let $(G, +)$ be an abelian group and define a function $\phi : G \to G$ by $\phi(a) = -a$. Prove that ϕ is an isomorphism from $(G, +)$ to $(G, +)$.

Problem 4 (4 points). Consider the group

$$G = \{a \in \mathbb{Q} : a = 2^n \text{ or } a = -(2^n) \text{ for some } n \in \mathbb{Z}\},$$

with the operation of multiplication. (You may assume that this is a group.)
Prove that G is not cyclic.

Problem 5 (4 points). For each part, make sure to justify your answer:

(a) Give an example of a subset of $\mathbb{R}$ that is closed under addition, but not closed under multiplication.

(b) Give an example of a subset of $\mathbb{R}$ that is closed under multiplication, but not closed under addition.

Problem 6 (4 points). Let G be the group $\{1, 3, 7, 9\}$ with the operation $\cdot_{10}$ of *multiplication* modulo 10. (You may assume this is a group.)

(a) List all of the subgroups of G.
How do you know that your list is complete?

(b) Draw the subgroup diagram of G.

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