MATH 120A MIDTERM EXAM (YELLOW PAPER)

WINTER 2015

Student name:

Student ID number:

Instructions

- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- When time is called, you must stop working immediately, close your exam, and remain seated until your exam is collected.
- If you want to leave your seat for any reason before time is called, raise your hand and remain seated until acknowledged.

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Problem 1 (10 points). Mark each statement 'T' for true (meaning always true) or 'F' for false (meaning sometimes false). You do NOT need to justify your answers to this problem.

T F If G is a group whose operation is associative, then G is called an abelian group.

T F Let $(S, *)$ be a binary structure and let $A, B \subseteq S$. If A and B are closed under $*$, then their union $A \cup B$ is closed under $*$.

T F The set of all nonzero 3×3 matrices with real entries forms a group under the operation of matrix multiplication.

T F The group $(\mathbb{C}^*, \cdot)$ of nonzero complex numbers under multiplication has a subgroup that is isomorphic to $\mathbb{Z}_4$.

T F If a group G has an element of order 2, then it also has a subgroup of order 2.

T F Any two groups of order 2 are isomorphic to each other.

T F Being abelian is a structural property of groups.

T F Let G be a group and $a, b, c \in G$. If $aba = aca$, then $b = c$.

T F Every cyclic group is abelian.

T F There is a bijection from $\mathbb{R}$ to $\mathbb{Q}$.

Problem 2 (4 points). Define the italicized terms:

(a) What is an *identity element* for a binary structure $(S, *)$?

(b) Assume that $(S, *)$ has an identity element and let $a \in S$.
What is an *inverse* of a in $(S, *)$?

Problem 3 (4 points). Let $n \in \mathbb{N}$ and recall that $GL_n(\mathbb{R})$ denotes the group of all invertible $n \times n$ matrices with real entries under the operation of matrix multiplication.
Let $\vec{x} \in \mathbb{R}^n$ and define the set

$$H = \{A \in GL_n(\mathbb{R}) : A\vec{x} = \vec{x}\},$$

where $A\vec{x}$ denotes the product of a matrix and a vector in the usual sense.
Prove that H is a subgroup of $GL_n(\mathbb{R})$.

Problem 4 (4 points). Let $(S, *)$ denote an arbitrary binary structure.
Prove that the property P saying

$$a * a = a \text{ for every } a \in S$$

is a structural property.

Problem 5 (4 points). For each part, make sure to justify your answer:

(a) Let $\mathbb{R}^*$ denote the set of all nonzero real numbers.
Give an example of a nontrivial subgroup of $(\mathbb{R}^*, \cdot)$ that is finite.

(b) Give an example of a proper subgroup of $(\mathbb{Q}, +)$ that is NOT cyclic.

Problem 6 (4 points). Consider the group $\mathbb{Z}_{12}$ with the operation of addition modulo 12.

(a) List all of the subgroups of $\mathbb{Z}_{12}$.
How do you know that your list is complete?

(b) Draw the subgroup diagram of $\mathbb{Z}_{12}$.

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