MATH 120B MIDTERM EXAM (WHITE PAPER)

SPRING 2015

Student name:

Student ID number:

Instructions

- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- When time is called, you must stop working immediately, close your exam, and remain seated until your exam is collected.
- If you want to leave your seat for any reason before time is called, raise your hand and remain seated until acknowledged.

1	/ 12
2	/ 4
3	/ 4
4	/ 4
5	/ 4
Total	/ 28

Problem 1 (12 points). Mark each statement 'T' for true (meaning always true) or 'F' for false (meaning sometimes false). You do NOT need to justify your answers to this problem.

T F Every infinite ring has characteristic zero.
T F If R is a ring, then there is a field F and an injective homomorphism from R to F.
T F There is an injective homomorphism from $\mathbb{Z}_6$ to $\mathbb{Z}$.
T F If R is a ring with unity, then the only idempotent elements of R are 0 and 1.
T F Let R be a ring with unity. Then x is never a zero divisor in $R[x]$.
T F The rings $2\mathbb{Z}$ and $3\mathbb{Z}$ are isomorphic to each other.

T F Let R be a commutative ring with unity $1 \neq 0$. If the cancellation laws hold in R, then R is a field.
T F Let R be a ring with unity. Every element of R is either a unit or a zero divisor.
T F If the rings R_1 and R_2 are isomorphic to each other, then they have the same characteristic.
T F If D_1 and D_2 are integral domains, then their direct product $D_1 \times D_2$ is an integral domain.
T F If R and R' are rings with unity and $\phi : R \to R'$ is a homomorphism, then $\phi(1_R) = 1_{R'}$.
T F There is a ring of characteristic 4.

Problem 2 (4 points). Give the requested definitions.

(a) What is Euler's phi-function φ?

(b) Let R be a ring with unity. What is a *unit* of R?

Problem 3 (4 points). Let R and R' be rings and let $\phi : R \to R'$ be a homomorphism.
(a) Prove that if R is a field then ϕ is either trivial (maps everything to zero) or injective.

(b) Give an example where R is not a field and ϕ is neither trivial nor injective.

Problem 4 (4 points). Compute 3^{70} mod 14. Show your work. Say what theorems you are applying, if any.

Problem 5 (4 points). Let K be a field and let $K(x)$ denote the field of quotients of the polynomial ring $K[x]$:

$$K(x) = \left\{ \frac{f(x)}{g(x)} : f(x), g(x) \in K[x] \ \ \& \ \ g(x) \neq 0 \right\}.$$

Let $\alpha \in K$ and define a subset S of $K(x)$ by

$$S = \left\{ \frac{f(x)}{g(x)} : f(x), g(x) \in K[x] \ \ \& \ \ g(\alpha) \neq 0 \right\}$$

where $g(\alpha)$ denotes $\phi_\alpha(g(x))$, the evaluation of $g(x)$ at α. Prove that S is a subring of $K(x)$.