MATH 120B MIDTERM EXAM (WHITE PAPER)

SPRING 2015

Student	name:

Student ID number:

Instructions

- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- When time is called, you must stop working immediately, close your exam, and remain seated until your exam is collected.
- If you want to leave your seat for any reason before time is called, raise your hand and remain seated until acknowledged.

1	/	12
2	/	4
3	/	4
4	/	4
5	/	4
Total	/	28

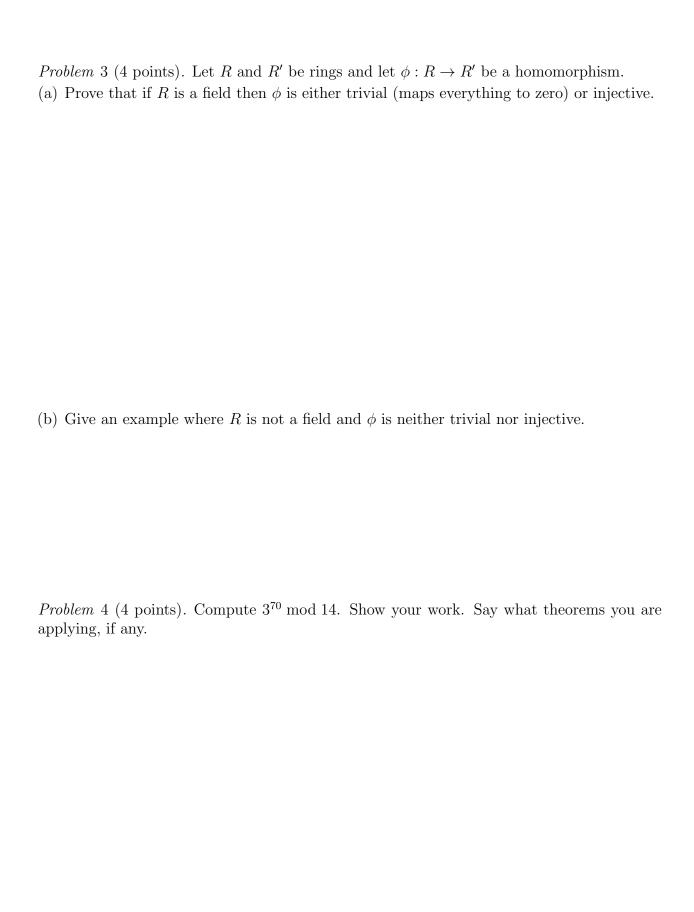
Problem 1 (12 points). Mark each statement 'T' for true (meaning always true) or 'F' for false (meaning sometimes false). You do NOT need to justify your answers to this problem.

- T F Every infinite ring has characteristic zero.
- T F If R is a ring, then there is a field F and an injective homomorphism from R to F.
- T F There is an injective homomorphism from \mathbb{Z}_6 to \mathbb{Z} .
- T F If R is a ring with unity, then the only idempotent elements of R are 0 and 1.
- T F Let R be a ring with unity. Then x is never a zero divisor in R[x].
- T F The rings $2\mathbb{Z}$ and $3\mathbb{Z}$ are isomorphic to each other.
- T F Let R be a commutative ring with unity $1 \neq 0$. If the cancellation laws hold in R, then R is a field.
- T F Let R be a ring with unity. Every element of R is either a unit or a zero divisor.
- T F If the rings R_1 and R_2 are isomorphic to each other, then they have the same characteristic.
- T F If D_1 and D_2 are integral domains, then their direct product $D_1 \times D_2$ is an integral domain.
- T F If R and R' are rings with unity and $\phi: R \to R'$ is a homomorphism, then $\phi(1_R) = 1_{R'}$.
- T F There is a ring of characteristic 4.

Problem 2 (4 points). Give the requested definitions.

(a) What is Euler's phi-function φ ?

(b) Let R be a ring with unity. What is a *unit* of R?



Problem 5 (4 points). Let K be a field and let K(x) denote the field of quotients of the polynomial ring K[x]:

$$K(x) = \left\{ \frac{f(x)}{g(x)} : f(x), g(x) \in K[x] \& g(x) \neq 0 \right\}.$$

Let $\alpha \in K$ and define a subset S of K(x) by

$$S = \left\{ \frac{f(x)}{g(x)} : f(x), g(x) \in K[x] \& g(\alpha) \neq 0 \right\}$$

where $g(\alpha)$ denotes $\phi_{\alpha}(g(x))$, the evaluation of g(x) at α . Prove that S is a subring of K(x).