MATH 120B MIDTERM EXAM (YELLOW PAPER)

SPRING 2015

Student name:

Student ID number:

Instructions

- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- When time is called, you must stop working immediately, close your exam, and remain seated until your exam is collected.
- If you want to leave your seat for any reason before time is called, raise your hand and remain seated until acknowledged.

1	/ 12
2	/ 4
3	/ 4
4	/ 4
5	/ 4
Total	/ 28

Problem 1 (12 points). Mark each statement 'T' for true (meaning always true) or 'F' for false (meaning sometimes false). You do NOT need to justify your answers to this problem.

T F There is a field of characteristic 4.

T F If D is an integral domain, then there is a field F and an injective homomorphism from D to F.

T F If p is a prime number and $a \in \mathbb{Z}$ is not divisible by p, then $a^{\varphi(p)} \equiv 1 \pmod{p}$.

T F If D and D' are integral domains and $\phi : D \to D'$ is a nontrivial homomorphism, then $\phi(1_D) = 1_{D'}$.

T F Let S be a set and let f be a function from S to S. If f is injective, then it is surjective.

T F If $(F, +, \cdot)$ is a field and F^* is the set of nonzero elements of F, then $(F^*, \cdot)$ is a group.

T F There is a surjective homomorphism from $\mathbb{Z}$ to $\mathbb{Z}_6$.

T F Let K be a field and let $f(x) \in K[x]$. If $f(\alpha) = 0$ for all $\alpha \in K$, then $f(x)$ is the zero polynomial.

T F If K is a field and $f(x), g(x) \in K[x]$ are nonzero then the degree of $f(x)g(x)$ is $\deg f(x) + \deg g(x)$.

T F If D is an integral domain then the polynomial ring $D[x]$ is an integral domain.

T F Let K be a field and let $f(x) \in K[x]$ be nonzero. The number of roots of $f(x)$ is equal to $\deg f(x)$.

T F Every finite integral domain is a field.

Problem 2 (4 points). Give the requested definitions.

(a) What is a *field*?

(b) What is an *idempotent* element of a ring?

Problem 3 (4 points). Let R and R' be rings with unity 1 and $1'$ respectively, let $\phi : R \to R'$ be a homomorphism, and let $a \in R$.

(a) Prove that if $\phi(1) = 1'$ and a is a unit, then $\phi(a)$ is a unit.

(b) Give an example where $\phi(1) \neq 1'$ and a is a unit, but $\phi(a)$ is not a unit.

Problem 4 (4 points). Compute 2^{41} mod 27. Show your work. Say what theorems you are applying, if any.

Problem 5 (4 points). This problem asks you to verify one of the steps in the construction of a field of quotients.

Let D be an integral domain and let D^* denote the set of nonzero elements of D. Recall that the relation $\sim$ on $D \times D^*$ defined by $(a, b) \sim (a', b') \iff ab' = ba'$ is an equivalence relation, and let $\frac{a}{b}$ denote the equivalence class of (a, b).

Define the set

$$F = \left\{ \frac{a}{b} : (a, b) \in D \times D^* \right\}.$$

(a) Prove that there is a (well-defined) operation $\cdot$ on F given by $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

(b) Give at least one reason why this operation $\cdot$ on F could not be defined if D had zero divisors.