MATH 13 MIDTERM EXAM

WINTER 2014

Student name:

Student ID number:

Instructions

- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- The time remaining will be written on the board periodically.
- You may hand in your exam and leave early, but please do not do this during the last 5 minutes of the exam period because it may disturb other students.

1	/ 9
2	/ 9
3	/ 6
4	/ 6
5	/ 6
6	/ 6
Total	/ 42

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Problem 1 (9 points).

(a) For $n = 1, 2, 3, \ldots$ define the interval $A_n = (-1/n, n)$. Rewrite the intersection of intervals $\bigcap_{n\in\mathbb{N}} A_n$ as an interval. For this problem you do not need to prove that your answer is correct.

(b) Give an example of sets A and B with the property that $\mathcal{P}(A \cup B) \neq \mathcal{P}(A) \cup \mathcal{P}(B)$, and show that your sets have this property. ($\mathcal{P}$ means "power set.")

(c) Make a truth table for the compound statement $((P \implies Q) \implies P) \implies P$. (You can call this compound statement S to save space in your table.)

Problem 2 (9 points). For each statement below, circle T or F according to whether the statement is true or false. You do NOT need to justify your answers.

T F $\sim(P \wedge Q)$ is logically equivalent to $(\sim P) \wedge (\sim Q)$.

T F For all sets A and B, $A \cap B \subseteq A \cup B$.

T F For all sets A, B, and C, if $A \cap B \cap C = \emptyset$ then $\{A, B, C\}$ is pairwise disjoint.

T F $(P \implies Q) \wedge (P \implies \sim Q)$ is a contradiction.

T F For all a, b, c, d, if $\{a, b\} = \{c, d\}$, then $a = b$ and $c = d$.

T F $\exists x \in S, (P(x) \vee Q(x))$ is logically equivalent to $(\exists x \in S, P(x)) \vee (\exists x \in S, Q(x))$.

T F $\emptyset \subseteq \emptyset$.

T F $(P \implies R) \wedge (Q \implies R)$ logically implies $(P \vee Q) \implies R$.

T F For all sets A, A_1, A_2, B, B_1, and B_2, if $\{A_1, A_2\}$ is a partition of A and $\{B_1, B_2\}$ is a partition of B, then $\{A_1 \times A_2, B_1 \times B_2\}$ is a partition of $A \times B$.

Problem 3 (6 points). Let $x \in \mathbb{Z}$. Prove that if $x^3 \equiv 1 \pmod{3}$, then $x^2 \equiv 1 \pmod{3}$.

Problem 4 (6 points). Let $x, y \in \mathbb{Z}$. Prove that if $4 \nmid x$ and $2 \mid y$, then $4 \nmid (x + y^2)$.

Problem 5 (6 points). PROVE or DISPROVE the following statement:
For all sets A, B, and C,

$$(A \subseteq C \wedge B \subseteq C) \iff (A \cup B \subseteq C).$$

Problem 6 (6 points). PROVE or DISPROVE the following statement:
There is a real number $x \in [0, 2]$ such that

$$2x^4 - 4x^3 + 1 = 0.$$

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