MATH 13 MIDTERM EXAM

WINTER 2014

Student name:

Student ID number:

INSTRUCTIONS

• Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
• Cheating in any form may result in an F grade for the course as well as administrative sanctions.
• The time remaining will be written on the board periodically.
• You may hand in your exam and leave early, but please do not do this during the last 5 minutes of the exam period because it may disturb other students.

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Problem 1 (9 points).

(a) For \( n = 1, 2, 3, \ldots \) define the interval \( A_n = [1/n, n] \). Rewrite the union of intervals \( \bigcup_{n \in \mathbb{N}} A_n \) as an interval. For this problem you do not need to prove that your answer is correct.

(b) Give an example of sets \( A \) and \( B \) with the property that \( \mathcal{P}(A) - \mathcal{P}(B) \nsubseteq \mathcal{P}(A - B) \), and show that your sets have this property. (\( \mathcal{P} \) means “power set.”)

(c) Make a truth table for the compound statement \((P \lor Q) \implies R\).
Problem 2 (9 points). For each statement below, circle T or F according to whether the statement is true or false. You do NOT need to justify your answers.

T F \( (P \Rightarrow R) \lor (Q \Rightarrow R) \) logically implies \( (P \lor Q) \Rightarrow R \).

T F For all \( a, b, c, d \), if \( \{a, b\} = \{c, d\} \) and \( a = c \), then \( b = d \).

T F For all sets \( A, B, C, D, \) and \( U \), if \( \{A, B, C, D\} \) is a partition of \( U \), then so is \( \{A \cup B, C \cup D\} \).

T F For all sets \( A \) and \( B \), \((A \times B) \cup (B \times A) = (A \cup B) \times (A \cup B)\).

T F \( \sim \forall x \in S, P(x) \) is logically equivalent to \( \forall x \in S, \sim P(x) \).

T F \( \emptyset \in \emptyset \).

T F \( P \Rightarrow (\sim P \Rightarrow Q) \) is a tautology.

T F For all sets \( A \) and \( B \), if \( A \cap B = A \) then \( A \subseteq B \).

T F \( \sim(P \lor Q) \) is logically equivalent to \( (\sim P) \land (\sim Q) \).
Problem 3 (6 points). Let \( x, y, z \in \mathbb{Z} \). Prove that the following sum of absolute values is even: \(|x - y| + |y - z| + |z - x|\).

Problem 4 (6 points). Let \( x \in \mathbb{Z} \). Prove that if \( 3 \nmid (x^2 + 2) \), then \( 3 \mid x \).
Problem 5 (6 points). PROVE or DISPROVE the following statement:
For all sets $A$, $B$, and $C$,

$$(A \subseteq C \lor B \subseteq C) \iff (A \cap B \subseteq C).$$