MATH 13 MIDTERM EXAM

WINTER 2014

Student name:

Student ID number:

Instructions

- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- The time remaining will be written on the board periodically.
- You may hand in your exam and leave early, but please do not do this during the last 5 minutes of the exam period because it may disturb other students.

1	/ 9
2	/ 9
3	/ 6
4	/ 6
5	/ 6
6	/ 6
Total	/ 42

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Problem 1 (9 points).

(a) For $n = 1, 2, 3, \ldots$ define the inverval $A_n = [1/n, n]$. Rewrite the union of intervals $\bigcup_{n \in \mathbb{N}} A_n$ as an interval. For this problem you do not need to prove that your answer is correct.

(b) Give an example of sets A and B with the property that $\mathcal{P}(A) - \mathcal{P}(B) \not\subseteq \mathcal{P}(A - B)$, and show that your sets have this property. ($\mathcal{P}$ means "power set.")

(c) Make a truth table for the compound statement $(P \vee Q) \implies R$.

Problem 2 (9 points). For each statement below, circle T or F according to whether the statement is true or false. You do NOT need to justify your answers.

T F $(P \implies R) \vee (Q \implies R)$ logically implies $(P \vee Q) \implies R$.

T F For all a, b, c, d, if $\{a, b\} = \{c, d\}$ and $a = c$, then $b = d$.

T F For all sets A, B, C, D, and U, if $\{A, B, C, D\}$ is a partition of U, then so is $\{A \cup B, C \cup D\}$.

T F For all sets A and B, $(A \times B) \cup (B \times A) = (A \cup B) \times (A \cup B)$.

T F $\sim \forall x \in S, P(x)$ is logically equivalent to $\forall x \in S, \sim P(x)$.

T F $\emptyset \in \emptyset$.

T F $P \implies (\sim P \implies Q)$ is a tautology.

T F For all sets A and B, if $A \cap B = A$ then $A \subseteq B$.

T F $\sim(P \vee Q)$ is logically equivalent to $(\sim P) \wedge (\sim Q)$.

Problem 3 (6 points). Let $x, y, z \in \mathbb{Z}$. Prove that the following sum of absolute values is even: $|x - y| + |y - z| + |z - x|$.

Problem 4 (6 points). Let $x \in \mathbb{Z}$. Prove that if $3 \nmid (x^2 + 2)$, then $3 \mid x$.

Problem 5 (6 points). PROVE or DISPROVE the following statement:
For all sets A, B, and C,

$$(A \subseteq C \vee B \subseteq C) \iff (A \cap B \subseteq C).$$

Problem 6 (6 points). PROVE or DISPROVE the following statement:
There are $a, b \in \mathbb{Z}$ such that

$$a^2 - b^2 = 2.$$

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