MATH 13 SAMPLE FINAL EXAM

WINTER 2014

Student name:

Student ID number:

Instructions

- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- The time remaining will be written on the board periodically.
- You may hand in your exam and leave early, but please do not do this during the last 5 minutes of the exam period because it may disturb other students.

1	/ 15
2	/ 5
3	/ 5
4	/ 5
5	/ 5
6	/ 5
7	/ 5
8	/ 5
Total	/ 50

Problem 1 (15 points). For each statement below, circle T or F according to whether the statement is true or false. You do NOT need to justify your answers.

T F Every countably infinite subset of $\mathbb{R}$ is well-ordered.
T F For all integers a, b, and c, if $a \mid 2b$ and $b \mid 2c$ then $a \mid 2c$.
T F If A is a set and $\mathcal{P}(A)$ is infinite, then A must also be infinite.
T F There is a bijection from the closed interval $[0, \infty)$ to the open interval $(0, \infty)$.
T F For all sets A and B, if there is a surjection from A to B and A is countable, then B is countable.

T F If R is a symmetric relation on the set A, then $(A \times A) - R$ must also be a symmetric relation on A.
T F There are sets A, B, and C and functions $f : A \to B$ and $g : B \to C$ such that f is not injective and $g \circ f : A \to C$ is injective.
T F For every set A, there is a bijection from $\mathcal{P}(A)$ to $A^{\{0,1\}}$.
T F If R is a reflexive relation on the set A, then R^{-1} must also be a reflexive relation on A.
T F If R_1 and R_2 are equivalence relations on the set A, then $R_1 \cap R_2$ must also be an equivalence relation on A.

T F For all sets A, B, and C, there is a bijection from $A \times (B \times C)$ to $(A \times B) \times C$.
T F $\sim(P \wedge Q)$ is logically equivalent to $\sim P \wedge \sim Q$
T F $\emptyset \subseteq \emptyset$
T F For all integers a, b, n with $n \geq 2$, if $ab \equiv 0 \pmod{n}$ then $a = 0 \pmod{n}$ or $b = 0 \pmod{n}$.
T F Every well-ordered set of real numbers has a least element.

Problem 2 (5 points). Let A and B be sets. Prove that the following equality holds:

$$(A \cup B) - (A \cap B) = (A - B) \cup (B - A).$$

Problem 3 (5 points). Is this compound statement a tautology? Justify your answer.

$$((P \implies Q) \wedge (\sim P \implies Q)) \implies Q$$

Problem 4 (5 points). Let $a, a', b, b', n \in \mathbb{Z}$ with $n \geq 2$. Prove that if $a \equiv a' \pmod{n}$ and $b \equiv b' \pmod{n}$, then $a + b \equiv a' + b' \pmod{n}$.

Problem 5 (5 points). Define a sequence $(a_1, a_2, a_3, \ldots)$ by $a_1 = 3$, and for all $n \in \mathbb{N}$,

$$a_{n+1} = \begin{cases} 2a_n & \text{if } n \text{ is odd} \\ 4a_n & \text{if } n \text{ is even.} \end{cases}$$

Prove that $a_n \leq 3^n$ for all $n \in \mathbb{N}$.

Problem 6 (5 points). Let A be a set and let $f : A \to A$ be a function such that $f(f(x)) = x$ for all $x \in A$. Define the relation R on A by

$$R = \{(x, y) \in A : y = x \text{ or } y = f(x)\}.$$

Prove that R is an equivalence relation.

Problem 7 (5 points). Prove that there is a bijection from $A \times A$ to $A^{\{0,1\}}$.

Problem 8 (5 points). Prove that there is a partition P of $\mathbb{N}$ such that

(1) P is infinite, and
(2) every element of P is infinite.

(Hint: You may use the existence of a bijection $f : \mathbb{N} \to \mathbb{N} \times \mathbb{N}$.)