MATH 13 SAMPLE MIDTERM EXAM

2014 FEB 10

Student name:

Student ID number:

Instructions

- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- The time remaining will be written on the board periodically.
- You may hand in your exam and leave early, but please do not do this during the last 5 minutes of the exam period because it may disturb other students.

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2	/ 9
3	/ 6
4	/ 6
5	/ 6
6	/ 6
Total	/ 42

Problem 1 (9 points). For the problems on this page, you should show your work, but you do not need to write proofs.

(a) Give an example of an element of the set $([1,2] \cup [3,4]) \times ([1,2] \cup [3,4])$ that is NOT an element of the set $([1,2] \times [1,2]) \cup ([3,4] \times [3,4])$.

(b) Write the set $\{x \in \mathbb{R} : x^2 > 1 \implies x > 1\}$ as an interval.

(c) Make a truth table for the compound statement $P \implies (Q \implies R)$

Problem 2 (9 points). For each statement below, circle T or F according to whether the statement is true or false. You do NOT need to justify your answers.

T F $(P \implies R) \vee (Q \implies R)$ logically implies $(P \wedge Q) \implies R$

T F $\sim \forall x \in S, P(x)$ is logically equivalent to $\exists x \in S, \sim P(x)$

T F $\forall x \in S, (P(x) \vee Q(x))$ is logically equivalent to $(\forall x \in S, P(x)) \vee (\forall x \in S, Q(x))$

T F $(P \implies Q) \vee (Q \implies P)$ is a tautology

T F $P \implies Q$ is logically equivalent to $\sim P \implies \sim Q$

T F For all sets A and B, if $A \cap B = A$ then $B \subseteq A$

T F For every set A, $\emptyset \subseteq A$

T F For all sets A and B, if $A \neq B$ then $(A - B) \cup (B - A) \neq \emptyset$

T F For all sets A and B, $(A \times B) \cap (B \times A) = (A \cap B) \times (A \cap B)$

Problem 3 (6 points). Prove or disprove the following statement: For all sets A, B, and C, $A - (B - C) = (A - B) \cup (A \cap C)$.

Problem 4 (6 points). Let $x, y \in \mathbb{Z}$.

Prove that $x \equiv y \pmod 6$ if and only if $x \equiv y \pmod 2$ and $x \equiv y \pmod 3$.

Problem 5 (6 points). Let $x, y \in \mathbb{Z}$.

Prove that if $x + y^2$ is odd, then so is $x^2 + y$.

Problem 6 (6 points). Let x, y, z be positive integers.

Prove that if $x^2 + y^2 + z^2 \geq 13$, then at least one of x, y, and z is ≥ 3.