Math 151 Midterm Exam
SAMPLE

Winter 2013

Instructions

Complete only FIVE of the EIGHT problems. If you do more, I will NOT identify your best work for you, but instead will grade an arbitrary subset of cardinality 5. Cross out anything that you do not want me to grade.

TIME LIMIT You have 5 hours to complete the exam from the moment you open it. Any writing on the paper done after the time limit has elapsed must be clearly indicated as such—if it turns out that the exam was too hard then you might receive partial credit for solutions written after time.

SOURCES You may use any books and notes that you wish, including any material posted online, provided that you cite your sources. You may use search engines to find such material. You may NOT collaborate with, give help to, or receive help from, any other person on any exam problems.

1. Prove that there is an injection from ω_1 to $\mathcal{P}(\omega)$, the power set of ω.
   Hint: you will need some form of the Axiom of Choice.

2. Give an example of three sets, all with the same rank. Justify your answer.

3. Are there two disjoint proper classes? Justify your answer.

4. Can the ordinal $\omega + 1$ be partitioned into two sets of the same order type? That is, are there disjoint sets A and B with $A \cup B = \omega + 1$ and $\text{type}(A; <) = \text{type}(B; <)$?

5. Show that if there is no injection from X to a proper subset of X, then X is finite. *Hint: you will need some form of the Axiom of Choice.*

6. Show that if α is an ordinal with $2 \leq \alpha \leq |2^\omega|$ then there is a bijection of α^ω with 2^ω.

7. Prove or disprove: every transitive set of ordinals is an ordinal.

8. Let R be a well-founded relation on a set A. Prove that there is no function $F : \omega \to A$ such that $F(n+1) \mathrel{R} F(n)$ for all $n < \omega$.