MATH 161 SAMPLE FINAL EXAM

SPRING 2014

Student name:

Student ID number:

Instructions

- Books, notes, and electronic devices may NOT be used. These items must be kept in a closed backpack or otherwise hidden from view during the exam.
- Cheating in any form may result in an F grade for the course as well as administrative sanctions.
- If you want to leave your seat for any reason, raise your hand and wait for permission.

1	/ 4
2	/ 4
3	/ 4
4	/ 4
5	/ 4
6	/ 4
7	/ 4
8	/ 4
Total	/ 32

1. Euclidean geometry

In this section, you may use the parallel postulate and its consequences.

Problem 1 (4 points).

(1) State Playfair's postulate.

(2) Define the reflection across a line ℓ.

Problem 2 (4 points). Circle 'T' or 'F' according to whether the statement is true or false. You do NOT need to justify your answers.

T F Given triangles $\triangle ABC$ and $\triangle A'B'C'$, if $\angle A \cong \angle A'$, $\angle B \cong \angle B'$, and $\overline{AC} \cong \overline{A'C'}$, then $\triangle ABC$ must be congruent to $\triangle A'B'C'$.

T F For any non-collinear points A, B, and C, there is a unique circle through A, B, and C.

T F If r_ℓ and r_m are reflections across parallel lines ℓ and m, we must have $r_\ell \circ r_m = r_m \circ r_\ell$.

T F For every isometry f and all points A and B we have $\overline{Af(A)} \cong \overline{Bf(B)}$.

Problem 3 (4 points). Suppose that the triangles $\triangle ABC$ and $\triangle A'B'C'$ are isosceles triangles with $AC = BC$ and $A'C' = B'C'$. Suppose that the bases $\overline{AB}$ and $\overline{A'B'}$ are congruent and the triangles $\triangle ABC$ and $\triangle A'B'C'$ have the same height. Prove that $\triangle ABC \cong \triangle A'B'C'$.

Problem 4 (4 points). Prove that if f is an isometry and $f \circ f$ is the identity function, then f has a fixed point.

Problem 5 (4 points). Let $\overline{AB}$ and $\overline{A'B'}$ be congruent line segments. Prove that there is one and only one even isometry f such that $f(A) = A'$ and $f(B) = B'$. (An isometry is called *even* if it is the composition of an even number of reflections.)

2. Hyperbolic geometry

In this section, assume the hyperbolic postulate instead of the parallel postulate.

Problem 6 (4 points).

(1) State the hyperbolic postulate.

(2) Define "Saccheri quadrilateral".

Problem 7 (4 points). Circle 'T' or 'F' according to whether the statement is true or false. You do NOT need to justify your answers. In hyperbolic geometry:

T	F	The sum of angles of every triangle is more than 180°.
T	F	If ℓ_1, ℓ_2, and ℓ_3 are lines, ℓ_1 is parallel to ℓ_2, and ℓ_2 is parallel to ℓ_3, then ℓ_1 must be parallel to ℓ_3.
T	F	There is an SAS similarity theorem.
T	F	Let f be a rotation (defined in terms of reflections as usual.) If f fixes an omega point, then f must be the identity function.

Problem 8 (4 points). Work in hyperbolic geometry. Let ℓ and m be parallel lines. Suppose that there are two distinct points P and P' on ℓ such that the perpendicular distance from P to m equals the perpendicular distance from P' to m. Prove that ℓ and m have a common perpendicular.