Problem 1 (6 points). Find the general solution of the following system of equations. If it is inconsistent, say so.

$$\begin{aligned} x_1 + x_2 + x_3 - x_4 &= 3 \\ x_1 - x_2 - x_3 - x_4 &= -1 \\ 2x_1 - 2x_4 &= 2 \end{aligned}$$

Problem 2 (6 points). Compute the following product of matrices.

$$\begin{pmatrix} 1 & -1 \\ 2 & 0 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$$

Problem 3 (6 points). Is the following matrix invertible? If not, why not? If so, what is its inverse?

$$A = \begin{pmatrix} 1 & 0 & 4 \\ 3 & 0 & 13 \\ 0 & 1 & 1 \end{pmatrix}$$

Problem 4 (6 points). Calculate the following determinant.

$$\begin{vmatrix} 3 & 0 & -1 & 2 \\ -1 & 0 & 3 & 3 \\ 1 & 2 & 5 & 0 \\ 2 & 0 & 0 & 1 \end{vmatrix}$$

Problem 5 (6 points). Consider the matrix

$$A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}.$$

Is A diagonalizable? If not, why not? If so, find a diagonal matrix D and an invertible matrix X such that $XDX^{-1} = A$.

Problem 6 (6 points). For each statement below, circle T or F according to whether the statement is true or false.

T / F For every invertible matrix A and every vector $\vec{x}$, if $A\vec{x} = \vec{0}$ then $\vec{x} = \vec{0}$.

T / F If A is a square matrix with real entries and λ is an eigenvalue of A, then the complex conjugate $\bar{\lambda}$ must also be an eigenvalue of A.

T / F If A and B are row-equivalent square matrices, then A and B must have the same eigenvalues.

T / F For all $n \times n$ matrices A and B, $\det(AB) = \det(BA)$.

T / F For every square matrix A, the transpose A^T has the same eigenvalues as A.

T / F For all $n \times n$ matrices A and B and all $i, j \in \{1, \ldots, n\}$, the (i, j) entry of AB is the (i, j) entry of A times the (i, j) entry of B.